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CONTENTS

HEAVY UNSTABLE PARTICLES

G.D. Rochester

The Physics Department, University of Manchester

1. Historical Introduction.

At the end of the war, Dr. Butler and the author reassembled the Blackett magnet and cloud chamber, with the object of investigating penetrating showers and looking for the negative proton by a process suggested by McConnell and Heitler. It was expected that unusual things might occur in penetrating showers because these are high-energy interactions in which mesons are formed. Towards the end of 1946 the shower shown in Figure 1 was obtained, in which there is a striking fork in the gas in the lower part of the chamber. Early in 1947 another unusual fork was found in a typical penetrating shower. In the first case the angle of the fork was 67° and the momentum of the charged particles about 300 MeV/c; and in the second case, the angle was 170° and the momentum of the particles about 700 MeV/c. After careful consideration, it was concluded that these events are most probably due to the decay in flight of unstable particles, the first photograph indicating the decay of a neutral particle and the second the decay of a charged particle. The main argument which led to this conclusion was the absence of similar events in the 3 cm. lead plate. It was not difficult to show that for any collision process the frequency should be some 500 times greater in the lead plate. Since only one or two similar events were found in the lead plate in several hundred photographs of penetrating showers, it was concluded that the forks must represent a process which depends upon the path length of the particle and not on the amount of matter which it traverses. This suggests that it must be a decay process. Detailed examination of specific processes supported this conclusion. For example, it was very unlikely that either of the forks could be an electron pair because of the large momenta and large angle between the tracks. Again, they could not represent the scattering of a single charged particle because of the absence of a recoil nucleus at the point of scattering. Finally, the angles completely excluded $\mu \rightarrow e$ decay and the (then unknown) $\pi \rightarrow \mu$ decay. Application of the Einstein mass-invariant relation led to minimum mass values of the order of 1,000 m_e, where m_e is the mass of the electron. It was thus clear that the particles were entirely new.

During the next two years the same equipment was run at sea level in Manchester, and no further examples of these particles were found. Then, in 1949 and early 1950, news came that Professor C.D. Anderson in Pasadena had obtained many examples of the neutral unstable particle and a few examples of the charged unstable particle. One of these neutral unstable particles is shown in Figure 2. From an examination of his photographs, Professor Anderson was led to confirm completely the reasoning which led to the original discovery. In consultation with Professor Blackett it was agreed to call these particles V-particles.

Late in the year 1949 the Manchester apparatus was moved to the Observatory on the Pic-du-Midi in the French Pyrenees, at a height of 2,867 m. Early in 1950 the cloud chamber was got working by Dr. Butler and his group and very soon many more examples of V-particles were obtained. Up to the present some 100 examples have been found by the Pic-du-Midi group, and many others by workers in different parts of the world; Fretter in Berkeley has more than 100

ely equal to the proton, and 4 negative with masses

examples, and Leighton (Pasadena), Thompson (Indiana), Astbury et al. (Manchester), McCusker (Dublin) and Bridge (Cambridge, U.S.A.), have many other examples. The characteristic features of the phenomena are the same. The Fretter photographs, taken in a multiplate cloud chamber, give very clear evidence that V-particles arise in nuclear interactions. An example of one of these is shown in Figure 3, the V-particle being to the left-hand side of the cloud chamber. The charged primary, which is presumably a nucleon, can be seen coming into the chamber and interacting in the fourth lead plate. Almost all of the photographs indicate that V-particles are produced in very high-energy interactions, certainly above 10^{10} eV.

The general results of all groups are to indicate that approximately one V^0-particle is produced in 25 interactions, and that the charged V-particles are only about 10% as frequent as the neutral V-particles.

II. Analysis of V^0-Tracks.

The first problem which arises in the analysis of V-tracks is the separation of neutral V-particles from charged V-particles. In general, a fairly way is to see whether the plane containing the tracks of the charged secondary particles contains the point of origin of the interaction. It seems that in general the secondary tracks are co-planar with the point of origin of the V^0-particle. This would indicate that neutral secondaries, if they exist at all, must be fairly light particles.

In conclusion, therefore, since the three types of secondary particles have been observed, the simplest decay schemes are the following:

$$V_1^0 \longrightarrow p^+ + \pi^-$$

$$V_2^0 \longrightarrow \pi^+ + \pi^-$$

111. Classification and Masses of V^0-Particles.

In order to get the most accurate value for the masses of the V^0-particles, it is important to use as much of the data as possible. This means that one has to try to separate the different types of V^0-particles by a study of the dynamics of the suggested decay schemes. A possible way of doing this has been suggested by Armenteros et al. Let p^* represent the momentum of a secondary decay particle in the rest system, and θ^* represent the angle which this track makes with the direction of the this decay have momenta above 1 BeV/c, and cannot, therefore, be definitely identified.

The usual method of identification of the products is an examination of the ionization and momentum of the tracks. A study of the theoretical ionization and momentum curves shows that, in the cloud chamber, it is possible to separate the known elementary particles below a momentum of about 700 MeV/c. The determination of the momentum of the secondary particle can usually be made with a fair degree of accuracy, but the estimation of ionization involves some uncertainty, since only visual estimations have been possible so far. The visual estimation of ionization is made difficult in the cloud chamber by the considerable difference in light intensity across the chamber and by the unavoidable variations in condensation throughout the volume of the chamber. This means that to make a fairly reliable estimate it is essential to have many tracks present in the chamber. It is unlikely that estimations of ionization are better than 50 %, and they may be in error by 100 %.

The results of the analysis of the secondaries of V^0-particles is to show that positive particles exist which are of approximately proton-like mass - as, for example, the case shown in Figure 6 - and observed decays, and when this is done the plot shown in Figure 9 is obtained. It is seen from this figure that the observed decays can be separated into the two main types.

It is now possible to calculate the masses of the V^0-particles assuming the masses of the proton and the π-meson. The formula which is used for this purpose follows from the application of the Einstein mass-energy relation, and is the following:

$$M^2 = m_+^{\,2} + m_-^{\,2} + 2p^+ p^- \left[\left\{ 1 + \left(\frac{m^+}{p}\right)^2 \right\}^{\frac{1}{2}} \left\{ 1 + \left(\frac{m^-}{p^-}\right)^2 \right\}^{\frac{1}{2}} - \cos\phi \right]$$

The results of the mass values of 12 V_1^0 particles are as given in Table 1, and the results for 8 V_2^0 particles are given in Table II.

It is interesting to calculate the Q values of the decay, and these are also shown in the tables. The Q value is defined as

$$Q = M - (m_+ + m_-)$$

The mean value of the mass of the V_1^0 particle is $(2203 \pm 12)\ m_0$, with a Q value of (46 ± 6) MeV. The mass of the V_2^0 is $(796 \pm 27)\ m_0$, if the decay is into approximately equal to π- or μ-mesons. Leighton et al. have found 18 heavily ionizing tracks which they identify as arising from 7 protons (or particles of slightly smaller mass), 9 negative mesons and 2 positive mesons. Thompson et al. have found one example of a positive proton secondary. It is thus clear that the secondary particles include three different types of elementary particle.

Before decay schemes can be written down, one must consider whether there is any evidence for other particles. One might enquire, for example, if there is any indication of interaction of the secondary particles with solid material in the chamber; or if there is any indication of neutral secondaries. The examination of the penetration of some 20 cm. of lead indicates that there is no evidence that any of the secondary particles suffer radiative loss like electrons. It is therefore very unlikely that any of the secondary particles are electrons. Again, there is evidence for nuclear interaction of secondary particles with an interaction mean free path of the order of 10 cm. This is consistent with some of the particles being protons or π-mesons.

It is difficult to establish clearly whether neutral secondaries exist or not. Perhaps the best be explained by the alternative types of decay of one V^0-particle of mass about 2600 m_e. The suggestion is attractive, since it involves a simplification, but so far the evidence seems to be against it. In the first place, there is no evidence for the existence of π^0-mesons in these decays. Several examples of V^0-particles decaying above lead plates in cloud chambers have been observed, and if these had produced π^0-particles, electronic showers should have been seen in the lead plates. Again, there is little evidence for the existence of the neutrons from the second decay scheme, although it should be stated that since the interaction length of the neutron is so long, it would be comparatively difficult to observe. Finally, one would expect the momentum spectra of the observed charged particles under decay scheme (2) to contain many more low momenta than decay scheme (1), since the neutron would carry away much more of the momentum. This is just the reverse of what is found. There are many more low momentum particles in V_1^0 decay products than in the V_2^0 decay products.

V^0-particle.

Again, let p^- and p^+ represent the momenta of the negative and positive secondary particles in the laboratory frame of reference, and assume that the tracks of these particles make angles with the initial direction of ϕ^- and ϕ^+; then one may define a quantity α which is equal to

$$\alpha = \frac{p^+ \cos\phi^+ - p^- \cos\phi^-}{p^+ \cos\phi^+ + p^- \cos\phi^-}$$

One can show that α is equal to

$$\frac{p_+^2 - p_-^2}{P^2} = \frac{m_+^2 - m_-^2}{M^2} + 2p^* \cos\theta^* \left\{ \frac{1}{M^2} + \frac{1}{P^2} \right\}^{\frac{1}{2}}$$

where P and M represent the momentum and mass of the V^0-particle and m^- and m^+ are the masses of the secondary products. It is found that the value of α is equal to (0.65 ± 0.02) for 16 cases of V_1^0 particles in which the secondary products can be shown to belong to the first decay scheme. Again, it is found that the value of α is equal to (0.05 ± 0.06) for 10 cases of V_2^0 decays. This is just what is expected from the formula. One can therefore examine the values of α for all of the case showed a negative particle which had a mass of about 1000 m_e.

An unusual case was also obtained by Armenteros et al. at the Pic-du-Midi, and it is shown in Figure 10. A heavily-ionizing negative particle comes out of an interaction in the lead plate and after traversing a few centimetres of gas decays into a lighter particle which is at minimum ionization. The ionization density of the short track is difficult to measure, but it is probably more than four times minimum. The angle between the two tracks is 100° and the momentum of the secondary charged particle is (180 ± 10) MeV/c.

In this same work another very unusual decay was obtained. This showed a charged particle which had penetrated the lead plate in the chamber and which decayed into a charged particle of about 100 MeV/c and presumably one or more neutral particles. One of the neutral particles decayed like a V^0-particle, lower down in the chamber. This photograph might give a hint as to the mechanism of decay of the charged V-particle.

Very recently, Dr. O'Ceallaigh* has found five

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* O'Ceallaigh, C. 1951. Phil.Mag. <u>42</u>, 1032.

π-mesons, Q value (122 ± 13) MeV. If the decay is into μ-mesons, the mass is $(705 \pm 32)\ m_e$.

IV. The Leighton Decay Scheme.

Leighton et al.* have suggested that the decay of the V^0-particles is three-body, and takes place according to the following schemes:

$$V^0 \nearrow \quad V_1^0 \longrightarrow p^+ + \pi^- + {}^{0} \qquad \ldots\ldots (1)$$

$$V^0 \searrow \quad V_2^0 \longrightarrow n^0 + \pi + \pi^- \qquad \ldots\ldots (2)$$

Leighton's suggestion is that V_1^0 and V_2^0 particles represent similar forms of decay into protons or neutrons and π-mesons. The Q value of the first decay scheme calculated from a two-body decay scheme involving charged secondaries only, would be about half of the Q value of the second scheme, if also calculated as a two-body decay. This is actually what is found, the observed ratio being about 2.6. Leighton also suggests that the Q value of the second form of decay would be about the correct value for a three-body decay. Both forms of decay would

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* Leighton, R.B., Wanlass, S.D., & Alford, W.L. Phys.Rev. <u>83</u>, 843, 1951.

It thus looks as though there may be different types of charged V-particles, as well as different types of neutral V-particles. The decay scheme for all the charged V-particles might be of the form:

$$V^{\pm} \; (\equiv \kappa^{\pm}) \rightarrow \mu^{\pm} + V_{2}^{o} + \nu^{o}$$

It is much too soon even to indicate what the final answer to these perplexing problems will be. A new field of cosmic-ray physics has been opened up, and it is certain that results of great interest and importance will be found during the next few years.

TABLE II

Mass values of 8 V_2^o particles

Catalogue no.	5	35	38	53	63*	66*	69	90*
α	-0.07	+0.38	+0.33	-0.03	-0.29	+0.51	+0.07	+0.05
Mass value (m_e)	796 ± 130	883 ± 50	872 ± 50	841 ± 60	820 ± 50	700 ± 30	673^{+100}_{-50}	785 ± 30
Q-value (Mev)	122 ± 65	165 ± 25	160 ± 25	144 ± 30	134 ± 25	74 ± 15	61^{+50}_{-25}	116 ± 15

*Events in which both secondary particles must have been less massive than the proton.

Fig. 6 A V^0-decay in which one of the secondaries can be identified as a proton. The negative particle is at minimum ionization and has a momentum of about 100/MeV/c; it is therefore probably a meson.
(After Armenteros et al).

Fig. 7. The frequency distribution of the momenta of the secondary particles.
(After Armenteros et al).

Fig. 8. The decay in flight of the negative secondary of a V_1^0 particle. This decay, which is ascribed to a $\pi^- \rightarrow \mu^-$ decay, can be seen in the middle of the chamber below the lead plate. The apex of the V^0-fork is above the lead plate.
(After Armenteros et al).

Fig. 9. The α-plot.
(After Armenteros et al).

Fig. 10. The decay of a slow charged V-particle. The charged V-particle has produced the short dense track from the end of which a fast charged particle emerges and moves horizontally across the chamber. Details of these particles are given in the text.
(After Armenteros et al).

V. Other Data about the V-Particles.

Careful examination of the points of decay of V-particles in relation to their points of origin, allowing for time dilation, has been made by Anderson and others, and it seems that the lifetime of both forms of V^0-particle are about 5×10^{-10} sec. This is a difficult measurement to make, and no more precise value of the lifetimes can be given at present. The frequency of the occurrence of both forms of V-particle seems to be about the same.

VI. Charged V-Particles.

It appears that about 10 % of all V-particles emerging from penetrating showers are charged V-particles. Here again, exact information about these particles is difficult to obtain unless the particle is heavily ionizing. Unfortunately, in the cloud chamber most of the particles are very fast and therefore it is not possible to identify them. It is clear, even from the early work, that the secondary charged products of charged V-particles are not electrons.

Barker, Butler and Rosser found several examples of slow charged particles, of masses intermediate between the π-meson and the proton: one very striking

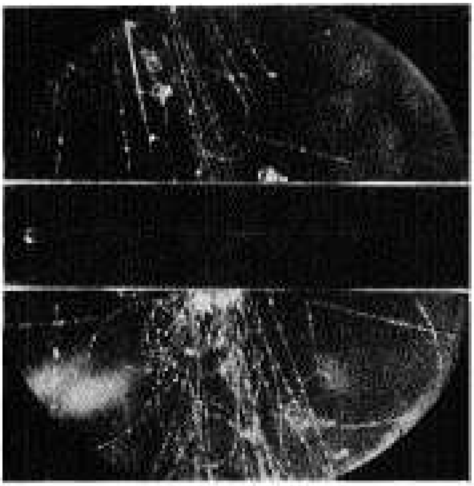

FIGURE 1

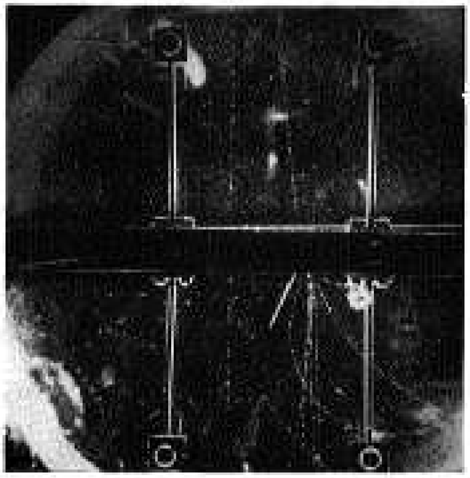

FIGURE 2

Rochester 1]

examples of the decay of charged particles in the photographic emulsion. The first one, which he calls K_1, has a measured mass of 1320 m_e and looks very similar to the decaying particle of Figure 10. The momentum of the decay product is 250 MeV/c. A second particle found by O'Ceallaigh, which he calls K_2 seems to be the decay of a particle of intermediate mass into a μ-meson, which later decays into an electron. This work suggests that charged V-particles and K-mesons may be examples of the same type of particle. It is difficult to assign decay schemes with certainty, but the cloud chamber work and one of Dr. O'Ceallaigh's mesons might indicate decay of the form

$$V^{\pm} \;(= K_{2\pm}) \rightarrow \mu^{\pm} + V_2^{o}$$

This decay scheme would give a mass for the V-particle of approximately 1250 m_e. This simple scheme cannot, however, explain all the cases of charged V's, nor the particle K_1 found by O'Ceallaigh. These particles require a three-body decay scheme, for example of the type

$$K_1^{\pm} \longrightarrow \mu^{\pm} + \nu^{o} + \nu^{o}$$

where ν^{o} represents the neutrino.

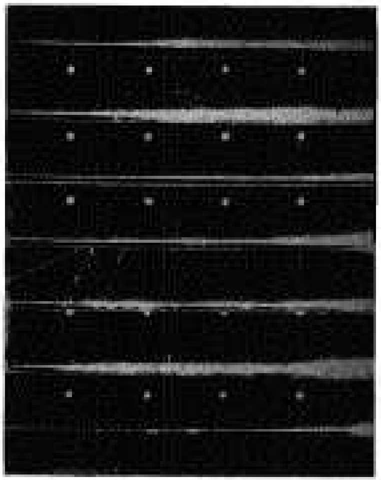

FIGURE 3

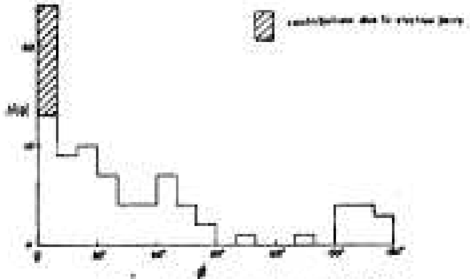

FIGURE 4

Rochester 1]

Description of Figures

Fig. 1. The first photograph of the decay of a neutral V-particle.

Fig. 2. Another example of the decay of a neutral V-particle
(After Seriff, Leighton, Hsaio, Cowan & Anderson).

Fig. 3. A striking photograph of a high-energy nuclear interaction in a lead plate, initiated by a fast charged particle, leading to the formation of a neutral V-particle. The decay of the V^0-particle can be seen to the left-hand side of the picture, below the fourth lead plate.
(After Fretter).

Fig. 4. The frequency distribution of the angles between the two branches of V-tracks.
(After Armenteros, Barker, Butler & Cachon).

Fig. 5. The decay in flight of a high-energy V_2^0 particle. The angle of the fork is 12° and the momenta are $p = 1.5$ BeV/c and $p^- = 1.6$ Bev/c. For this event $\alpha = -0.03$, leading to a mass of $(841 \pm 60)\ m_0$.
(After Armenteros et al).

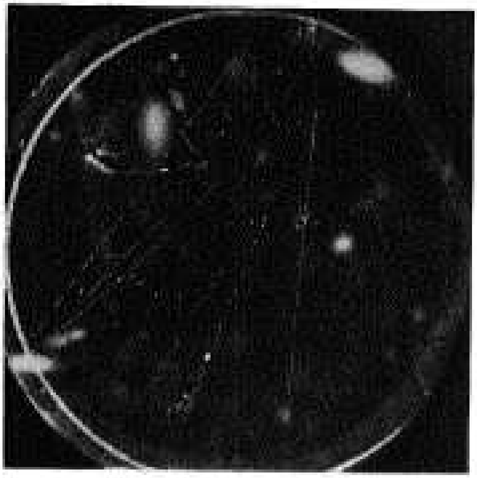

FIGURE 5

FIGURE 6

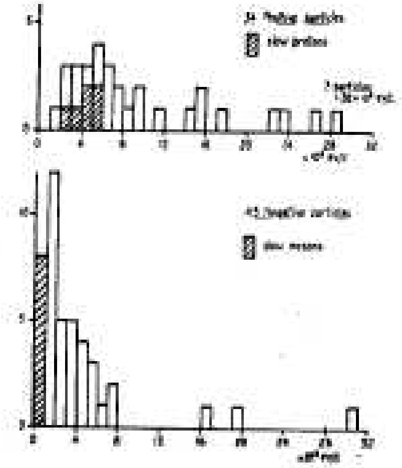

FIGURE 7

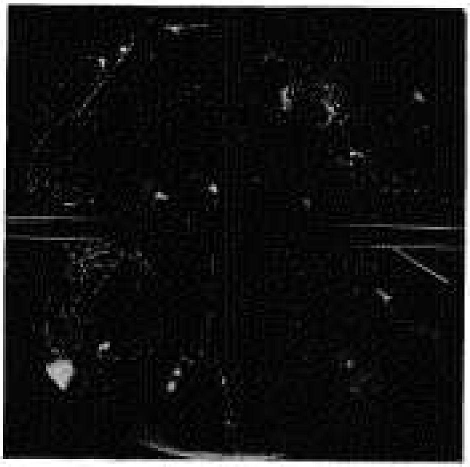

FIGURE 8

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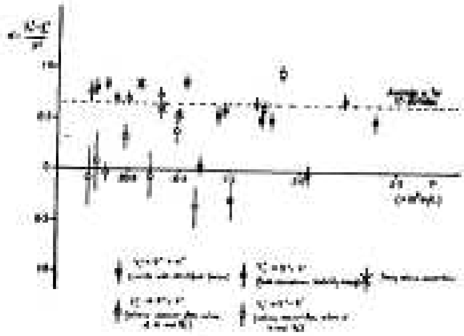

FIGURE 9

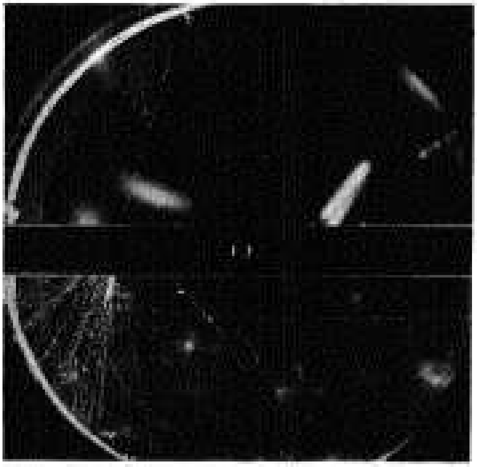

FIGURE 10

Rochester 1]

THE EXISTENCE AND PROPERTIES OF HEAVY CHARGED MESONS

C. O'Ceallaigh
H.H. Wills Physical Laboratory, Bristol
and University College, Cork.

For some years on the bases of certain isolated observations, the existence has been surmised of heavy unstable particles of mass greater than that of the π-meson, but only within the past eighteen months has the correctness of these speculations been demonstrated by unequivocal experimental proof.

The first suggestion that heavy charged particles of mass $\sim 1000\ m_0$ are to be found in cosmic radiation was put forward as long ago as 1944 by Leprince-Ringuet and Lhéritier[1]. This was based on the analysis of a collision between such a particle and an electron in a cloud-chamber photograph taken at the Largentière high-altitude laboratory. In 1947, Rochester and Butler[2] published a remarkable analysis of two events found in cloud-chamber photographs in association with penetrating showers. They interpreted these as examples of the decay in flight of heavy

firm the existence and establish the mode of decay of the τ-meson. Further examples of the decay of heavy charged mesons have been found by the Pasadena and Manchester groups, and also by other workers[9], but no direct measurement of mass could be made. Recently however, at Bristol[10], examples have been found in photographic plates of the decay to one charged particle of a heavy charged meson. In these cases conditions were such that it was possible to make direct measurements of the masses of the decaying particles. In addition, in one case, it was also possible to identify the charged secondary particle and to measure its energy accurately.

Since it will not be possible to cover the whole field of heavy mesons, charged and uncharged, the lecture will be confined to a review of the present state of the experimental evidence for the existence and properties of heavy charged mesons. Special emphasis will be laid on the results found by means of the photographic plate, the technique which seems to be best adapted to the problem, and that which is likely to prove most fruitful.

A brief digression will be made to describe the experimental procedures involved in making the necessary

are resolved, it is customary to measure the grain-density, defined as the number of developed grains per unit length of track. It is necessary to fix on a convention to deal with occasional cases of unresolved grains, and to minimise the inclusion of background grains. For a considerable range of low to moderate ionization in the photographic emulsion, there exists a linear relationship between ionization and g [12].

The relation between energy-loss per unit path, charge and velocity βc of an ionizing particle of charge Ze is given by the well-known Bethe-Bloch formula

$$- \frac{dT}{dx} = \text{constant} \times N z^2 \beta^{-2} \left[C - \beta^2 - \ln B^2 \beta^2 \right] \qquad (1)$$

Here N = the number of atoms per unit volume of the medium, C is a constant which depends on maximum possible energy transfer per impact, and also on the effective ionization potential of the struck atom, and B is written for the quantity $(1 - \beta^2)^{-\frac{1}{2}}$. This expression requires modification for condensed media, but the theory of the connection between grain density and ionization-energy-loss has not yet been fully worked out for the photographic emulsion. For

Then, we have $M_2/M_1 = R_2/R_1$.

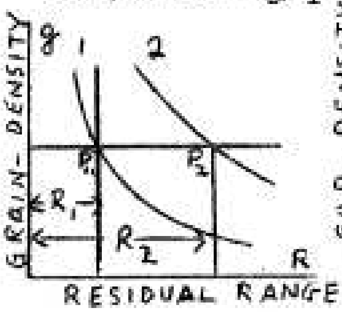

Fig 1

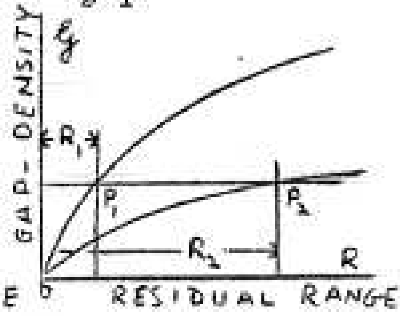

Fig 2

Sometimes these curves are given in the integrated form of total grain-count vs. R or total gap-length vs. R and present the appearance shown in Figs. 3 and 4. for linear plots.

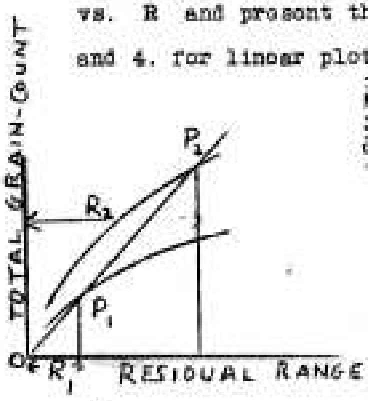

Fig 3

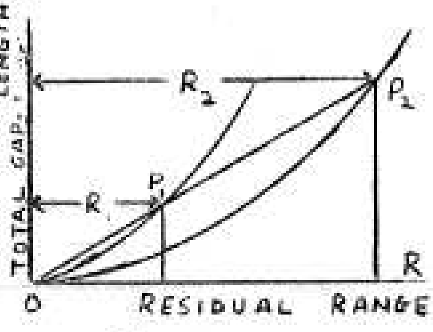

Fig 4

For this method of setting out the results, any straight line through the origin which cuts one curve, will, on production, cut the other curve (s) in corresmesons, one charged and one neutral. The measured momenta assuming reasonable decay schemes, indicated minimum masses for each type of unstable particle of the order 1000 m_o. Later, in 1948, Leprince-Ringuet and his collaborators[3] published a photomicrograph of a double star the smaller of which appeared to have been caused by a π^--meson ejected from the larger. The larger star appeared to have been caused by a charged particle the mass of which, computed from the estimated energy-release in the star, was at least 700 m_o. In 1949 a very remarkable event was found at Bristol[4] in the first exposure of the Kodak N.T.4. electron-sensitive emulsion. This was interpreted as the decay, at rest, of an unstable charged particle, the τ-meson, of mass ~ 1000 m_e. Three charged particles were produced in the primary decay. Of these, one was identified as a π^- meson because it came to rest in the emulsion giving rise to a characteristic σ -star.

Within the past eighteen months, measurements both by Anderson and his collaborators at Pasadena[5], and by the Manchester group[6] have confirmed the original interpretation of Rochester and Butler, and recent examples found at Bristol[7] and London[8] coneach position of the stage, the position of the track is read off on an eyepiecescale. In this way, we find a set of numbers which are the y-coordinates of the track corresponding to values of x separated by intervals of s microns. The interval s is known as the cell-size. Let these coordinates be $y_0, y_1 \ldots\ldots y_j$. (Fig. 5).

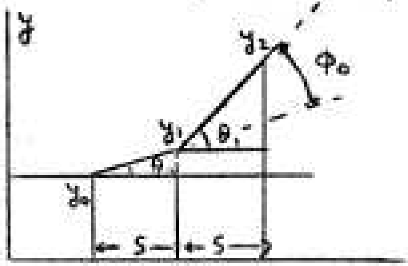

Fig 5

The set of second differences of the y_j is calculated from the eyepiece readings. We have $D_j = y_{j+2} - 2y_{j+1} + y_j$. Then, referring to the diagram, it will be seen that

$$\frac{T\,D_j}{s} = \vartheta_{j+1} - \vartheta_j = \phi_j \qquad (3)$$

T is the reduction factor which converts to radians, the angles y_j/s. Clearly ϕ_j is the angle between adjacent chords drawn to the track at points defined estimates of mass and energy. As some of them are recent and not readily accessible it is hoped that the references to the literature will be of use to those who desire further technical details.

Measurement of Mass and Energy of Ionizing Particles.

The three principal techniques used to estimate the mass and energy of ionizing particles in photographic emulsions, involve simultaneous measurement of

(1) Ionization and residual range.

(2) Multiple scattering and residual range.

(3) Ionization and multiple scattering.

The estimate of ionization is made by grain counts or gap-counts. Which is the more convenient depends on circumstances. Particularly in the case of the electron-sensitive emulsions, it is not possible to count the individual grains rendered developable by slow particles as owing to the high ionization the individual grains run together. In such cases, it is best to measure the fraction of the length of the track which is free from developed grains. This is known as the gap density (G) and has been studied by Hodgson[11]. For faster tracks where most of the individual grains variable ϕ about its mean value 0. Other measures of dispersion, of course, exist, for example the mean deviation μ defined as

$$\mu = \frac{1}{2\pi\sigma} \int_{-\infty}^{+\infty} |\phi| \exp\left[-\tfrac{1}{2} (\phi^2/\sigma^2) \right] d\phi \quad (5)$$

For a normal distribution $\mu = 0.7979\,\sigma$.

Now it may be shown that[17] $\sigma = \dfrac{K s^{\frac{1}{2}}}{p\beta c}$ (6)

K is known as the scattering constant, and its value may be calculated from theory using the chemical composition of the emulsion. Its value has also been found experimentally[13,18] using protons, mesons and electrons of known energy. We see that the reciprocal of the standard deviation of the distribution of ϕ is a measure of the energy of the particle which produces the track. In fact, any measure of the dispersion of ϕ may be used to measure the energy of the particle, provided we use the appropriate value of K. In practice, because of ease of computation, it has been customary to use the reciprocal of the mean deviation for this purpose.

So far we have supposed that our track is of such length that we are dealing effectively with the population, so that the particle's energy can be comparative measurements, however, it suffices to point out that ionization, g and G, depend only on the value of z and not on the mass of the particle which gives rise to the track. It is true that the value of G in (1) is weakly dependent on the mass of the particle, but as between particles of mass $\gg m_e$, this variation may be neglected. It is of interest to note that for $\beta \leqslant 0.9$, the equation (1) may be written in the simple form $-\frac{dT}{dx} = A\ \beta^{-y}$ where the values of the constant and exponent depend on the medium. For singly-charged particles, we have approximately,

	A	y
Aluminium	1.44	1.64
Emulsion	1.47	1.46

$-dT/dx$ being expressed in MeV/gram cm^2. Since dT/dx is a function of β and z only, it follows that particles of the same charge and velocity have residual ranges in direct proportion to their masses.

Consider the plots of g vs. R and G vs. R for two particles of mass M_1 and M_2 ; $M_2 > M_1$. The curves have the general appearance shown in Figs. 1 and 2. The points P_1 and P_2 may be called 'corresponding points', and are points for which particles of the same charge, have the same velocity.

values of ϕ_j contained in the sample. Each track yields $L/s - 1 = \nu$ values of ϕ. About 90 %, say n, of these may be taken to be statistically independent. The mean of the sample, will not, in general, be zero. For sufficiently large N the individual values of the mean computed for identical tracks, will be distributed normally about zero with standard deviation $= \sigma \overline{n}^{\frac{1}{2}}$.

The estimate of dispersion commonly employed has been termed $\bar{\alpha}$, and has the merit of extreme ease of computation, being the mean value of the individual ϕ_j all taken with positive sign.

$$\bar{\alpha} = 1/\nu \sum_{\nu = 0}^{\nu} |\phi_j| \qquad (7)$$

Stated in words, this quantity is the estimate of mean deviation from assumed mean zero. Now, if for a large number of tracks all of equal length and energy, the experimental values of $\bar{\alpha}$ be plotted, they will be found to be distributed about the population mean deviation μ in an asymmetrical manner, especially for small values of n. For larger values of n, the distribution of $\bar{\alpha}$ about μ will tend towards the normal. However, the estimate of energy derived from such samples will be based on $\bar{\alpha}^{-1}$, and will be ponding points. The data are sometimes plotted on log.-log. paper, in which case it may be shown that the corresponding points are given by the points of intersection of the curves with the family of straight lines which make an angle of 45° with either axis.

Multiple Scattering

Due to interactions with the atoms of the medium, the path of a charged particle through matter is tortuous, and the departures from straight-line motion may be used to estimate the value of $p\beta c$ proper to the particle where p = momentum = $M_0 B \beta c$, and M_0 is the rest-mass of the particle. The technique of energy measurement by multiple scattering has been largely developed by workers at Bristol[13)], Brussels[14)] and Manchester[15)]. There are several methods of carrying out this measurement. Of these, that described by Fowler[16)] is the most convenient in practice. Briefly the plate is set up on a microscope the stage of which is designed to have negligible departure from straight-line motion along two mutually perpendicular axes. The plate is 'lined up' so that the mean direction of motion corresponds to, say, the x-axis. By means of a micrometer screw, the stage is traversed along the x-axis by equal steps of s microns. At

Particle	P
π-meson	5.10
Proton	4.27
Deuteron	3.98
Triton	3.80

Mass Measurements by Technique 3. - g vs. $\bar{\alpha}$.

This method[16] is of great value for identifying particles which do not come to rest in the emulsion, and has been used with success to establish the nature of the 'shower' and other particles emitted from cosmic ray stars. These results are shown in Fig. 7, from which the method will readily be understood. In the light of recent work[20], these curves require modification in the region of minimum ionization. Instead of reaching constant grain-density as the energy increases, the curve proper to each particle first passes through a minimum value before reaching the final constant value or 'plateau' of grain-density.

τ -Mesons.

To the time of writing, six examples of the τ -decay have been observed, three at Bristol[4,7] and three at Imperial College, London[8,11]. Of these, four are suitable for measurement. The main by the cell-size s. For a track of length L microns, there will be $L/s - 1$ such values of ϕ_j.

Suppose that we have available a hypothetical track of constant energy and unlimited length such that the number $L/s - 1$ is very large. Then, on plotting the experimental distribution of ϕ_j, we would obtain a result such as that shown in Fig. 6.

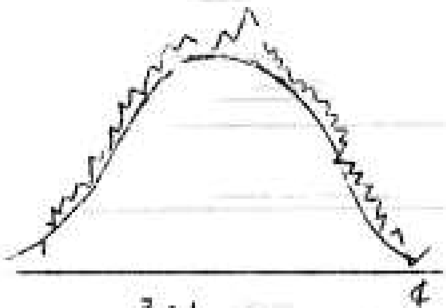

Fig 6

We call this distribution the 'population' and the individual members will be distributed approximately normally with mean zero. We shall suppose that it is normal. A normal population is characterised by the two-parameter equation

$$p(\phi)\, d\phi = \frac{1}{\sqrt{2\pi}\,\sigma} \exp - \left[1/2 \left(\frac{\phi - \bar{\phi}}{\sigma} \right)^2 \right] d\phi \quad (4)$$

where $\bar{\phi}$ is the mean, and σ is the standard deviation. In the present case $\bar{\phi} = 0$. The standard deviation σ is a convenient measure of the dispersion of the particles. It is likewise for the London events, so that it will not be necessary to describe them in detail.

Identification of the Particles and the Q-Value of the Reaction.

Since it seems certain that one of the decay particles in event 1 is a π-meson, it remains to discuss the evidence for the nature of the remaining pair of particles and of the three particles in event 2. Event 2 possessed the most favourable geometry so that it will be chosen for detailed description of the measurements.

It is a fortunate feature of this event that the track a is of great length (6400 μ), and that it could be demonstrated by gap-counting that it had almost come to rest before passing out of the emulsion. An estimate of mass by Technique 3, g vs. $\overline{\alpha}$, gave a value 285 $\pm$ 20 m_e, so that (a) is almost certainly a π-meson. Furthermore, since the uncertainty in range was only $\sim$ 400 μ, good values for the energy and momentum of the particle could be obtained by use of the range-energy relation. The values found were T = 19 $\pm$ 0.4 MeV and p = 75.4 MeV/c. Now, since the tracks a,b and c were shown to be coplanar, we may apply the conservation law to find the momenta of b and c. They were 85.8

measured by determining experimentally the value of the dispersion. We are constrained, in practice, however, to deal with tracks of finite length such that $L/s - 1$ has quite small value. Thus, our tracks constitute merely random samples drawn from the population. It might be thought that the value of $L/s - 1$ could be increased by decreasing s, but such reduction cannot be continued indefinitely. This is because for very small s, the dispersion is no longer a measure of the dispersion of ϕ, but rather of the unavoidable errors of observation coupled with those arising from imperfections in the motion of the stage. The conjoint effect of these is usually termed 'noise'. Thus the minimum acceptable value of s for a track of given energy is such that the dispersion due to scattering say σ true, is large compared with that due to 'noise'.

As will be seen from (b), σ_{true} increases as $s^{\frac{3}{2}}$. Techniques[16] exist for the elimination of 'noise'. They involve the measurement of scattering using two cell-sizes.

Since, in general $L/s - 1$ will be small, it follows that we can only make an estimate of the values of the mean, the standard deviation of ϕ from the

TABLE I

Total Kinetic Energy of the Decay Particles

Event	Assumed to be π-particles			
	Particle (a)	Particle (b)	Particle (c)	Total
1	1.04±0.10 MeV.	31±4 MeV.	33±4 MeV.	65± 8 MeV.
2	50±7.5 MeV.	13±2 MeV.	22±3 MeV.	85±15 MeV.
3	19±0.4 MeV.	24.2±2 MeV.	32±2 MeV.	75± 5 MeV.
				Weighted Mean 73.5±4 MeV.

TABLE II

Absolute or Relative Values of Mass of Secondary Particles

Event	Particle (a)	Particle (b)	Particle (c)
1	π = 274m_e	280±30	1.02±12%
2	1	1.10±15%	1.0 ±10%
3	285±20m_e	0.88±11%	1.03± 5%

TABLE III

Mass of Primary Particle in m_e

Particle	Authors	Length of track	$\mathcal{I} \sim$ R	g.d $\sim$ R	Conservation of momentum	Mode of decay
τ	Brown et al.	3100 μ	990±270	1080±160	949±16	$\tau \rightarrow \pi + \pi + \pi$
τ	Harding	740 μ	-	-	989±30	"
τ	Fowler	2070 μ	1015±280	910±220	969±10	"

distributed differently from those of $\bar{\alpha}$. The standard deviation S_E of the estimate of energy will depend on n. For large n, it will tend towards $\frac{1}{\sqrt{2N}}\ \sigma^{-1}$ [19].

By use of this expression, we may arrive at an approximate value for the statistical uncertainty of the estimate of energy deduced from measurements of scattering on a track of finite length measured in cell-size = s.

Mass Measurements by Technique 3. Multiple scattering vs. residual range.

A convenient and rapid method of estimating the masses of particles which come to rest in the emulsion, has been described by Menon and Rochat[13]. Briefly, the residual range R is first measured, and then an estimate of $\bar{\alpha}$ is made using, however, only the faster half of the track. This special $\bar{\alpha}$ is distinguished by the symbol $\hat{\alpha}$. They define as follows a parameter P

$$P = 1.37 \log R - 2.37 \log \hat{\alpha} \quad . \qquad (8)$$

This parameter may be shown to be a linear function of the logarithm of the mass value. R is measured in microns and $\hat{\alpha}$ in ^{o}/100 microns. Typical values of P for G.5 emulsions are secondary particle, which had a range of 2200 μ before it left the emulsion, was estimated by multiple scattering to be 250 MeV. This is about twice the rest-energy of the μ-meson, and the length of the track was such that this value could not be explained as arising from any reasonable statistical fluctuation in the measurement of $\bar{\alpha}$. The grain-count of the track was indistinguishable from the minimum value found by measurement of particles in the same region of the plate. Taken in conjunction with the above value of $\bar{\alpha}$, this observation showed that it was highly improbable that the mass of the particle exceeded 400 m_o. Thus, in terms of known particles, it could be a π or μ-meson or an electron. Because of these features, the primary particle which had a visible range of 4200 μ was subjected to careful scrutiny, and an estimate of its mass by Techniques 1 and 2 gave the following values; G vs. R 1350 $\pm$ 170 m_o and $\bar{\alpha}$ vs R, 1260 $\pm$ 290 m_o. A photomicrograph of the event is shown in Plate III, and the experimental gap-length curves (integral) in Fig. 6.

A second event found a short time later had even more remarkable characteristics and is shown in Plate IV. Superficially, it has the appearance of a π - μ - e decay, but the range of the intermediate particle features of the process will be visible from the photomicrographs in Plates I and II. Plate I shows the Bristol 1949 event already referred to. The direction of motion of the heavily-ionizing track τ is towards P as shown by the progressive increase of multiple scattering and grain-density. The appearance of the track in the neighbourhood of P is consistent with the assumption that it decayed at P to three charged particles a, b and c. Of these b and c are fairly fast as shown by the clear resolution of the separate grains which have grain-density of about $2\ g_{min.}$. That marked a is slow, and coming to rest in the emulsion, decays to a characteristic two-prong σ -star. This event provides the strongest evidence that (a) is the track of a π -meson. Furthermore, to within the error of measurement $\sim 2^{\circ}$, the three tracks a, b and c are coplanar. This provides strong independent evidence that we are dealing with the decay of the particle τ while at rest or moving with negligible momentum.

Plate II shows the second such event observed at Bristol. The main features agree with those of τ_1, including the coplanarity of the decay products a, b and c, the only point of difference being the more even partition of the available energy between the

Table IV

Particle	PRIMARY PARTICLE			SECONDARY PARTICLE						
	Length of track	Mass (m_e) α vs R	Mass (m_e) G vs R	Length of track	G.D.	$\frac{\alpha}{100\,\mu}$	Identity	$p\beta c$	Energy	Remarks
K 1	4100 μ	1260 $\pm$290	1350 $\pm$180	2200μ	$g_{plateau}$	0.10 $\pm$ 0.014°	$m_s < 400\ m_e$	250 MeV	182 MeV (assumed μ-meson)	Secondary charged particle could be a π-meson, μ-meson or electron.
K 2	5670 μ	1125 $\pm$260	1125 $\pm$150	1100μ	-	-	μ-meson 200 m_e < m_s < 300 m_e from α vs R	11.5 MeV	5.92 MeV	Secondary has a minimum track, at the end of its range and has therefore been identified as a μ-meson (decaying into an electron).
K 3	900 μ	1800 $\pm$1000	1800 $\pm$1000	150μ	$g_{plateau}$	-	-	-	-	
K 4	534 μ	-	-	3900μ	$g_{plateau}$	0.16$\pm$ 0.014°	$m_s < 350\ m_e$	145 MeV	95 MeV (assumed μ-meson)	Primary track is too steep for an accurate measurement of mass. Scattering measurements show that it is approaching the event and is of near-protonic mass. Secondary could be a π, μ or e.
K 5	2100 μ	1370 $\pm$320	-	171μ	$g_{plateau}$	-	-	-	-	Secondary track is short and steep. It appears to have a grain density ~ plateau.

$\pm$ 1 MeV/c and 96.3 $\pm$ 1 MeV/c respectively. These values of momentum, taken with the observed grain-densities, yielded estimates of mass $m_b = 240 \pm 30$ m_e and $m_c = 280 \pm 15$ m_e. Because the track b dips steeply in the emulsion, it is believed that the above value of m_b is an underestimate. These results make it most reasonable to suppose that all three decay products are π-mesons. If this is assumed to be the case, the energy of b and c may be calculated from the momenta, and the total energy release in the reaction: the Q-value may be computed. For the case under discussion, this was found to be 75 $\pm$ 5 MeV.

Analysis similar to the foregoing has been carried out on event 1 and on one event described in detail by Harding[8], leading to the Q-values given in Table I and to a weighted mean value of 75 $\pm$ 4 MeV. Assuming decay to three π-mesons of mass 274 m_e this value of Q fixes the mass of the τ-particle as 966 $\pm$ 8 m_e. The correctness of these conclusions rests on the assumption that the τ-meson decays at rest to three π-mesons, and that no neutral particle is emitted in the process. It has been calculated that the emission of a neutrino of energy $\sim$ 10 MeV would produce an average departure from coplanarity of $\sim 3^\circ$

one pion and one lepton and (c) two leptons. We will write λ for a neutral lepton which may be a photon or a neutrino. Scheme (a) demands that K be a fermion. Schemes (b) and (c) may be satisfied by a boson or a fermion by suitable choice of λ. We have,

$$K \rightarrow 2\pi_0 + \mu \qquad \text{(a)}$$

$$\rightarrow \pi_0 + \lambda + \mu \qquad \text{(b)}$$

$$\rightarrow 2\lambda + \mu \qquad \text{(c)}$$

Applying the conservation laws to the case K_1, we find the following minimal values of Q the energy-release and m_K the kappa mass.

Table V

Decay Scheme	$Q_{min.}$ (MeV)	$m_{min.}$ (m_e)
(a)	294	1310
(b)	349	1155
(c)	452	1090

These minimal masses are in good agreement within a possibility not excluded by the measurements. However five cases of τ-decay have now been observed in which the product particles are coplanar to within 2° and it does not appear reasonable to assume in all of these cases, that the neutral particle in a four-body decay has an exceptionally low energy and momentum. Further, the conclusion that the three charged particles had equal mass was tested by an independent argument based on the measurement of the angles between their initial directions of motion. The results given in Table II are favourable to this assumption.

Finally, the masses of the primary particles were estimated by application of the Techniques 1 and 2, with the results given in Table III.

A measurable case of τ-decay described by Hodgson[21] yielded three tracks coplanar to within 2°. Assuming that these three particles were π-mesons, energy estimates based on grain-counting gave a Q-value of 73.5 $\pm$ 7 MeV in agreement with the value adopted above.

Kappa Mesons.

During an investigation[22] of the μ-meson electron decay spectrum, an event was found in which a particle coming to rest in the emulsion decayed to a particle at minimum grain-density. The value of $p\beta c$ of the

Table VI

$m_K = 966\ m_e$

Scheme	$p\beta c$ of μ-meson
(a)	125 MeV
(b)	193 MeV
(c)	215 MeV

To the time of writing, the secondary particle from K_1 has the highest measured value of $p\beta c$ namely 250 MeV, but the standard deviation of this observation may be calculated to be ~ 17 %. Thus, while apart from considerations based on spin conservation, we may reject scheme (a), the <u>experimental</u> value $p\beta c$ = 250 MeV is not inconsistent with decay-schemes (b) and (c) if we assume $m_\tau = m_K$. It must be pointed out however, that we have at present no evidence that 250 MeV is approximate to the maximum $p\beta c$ of the charged secondary decay spectrum. Hence, while at the moment the evidence from this source is inconclusive, the finding of a few charged secondary particles even of moderate length, having values of $p\beta c$ in the range 200 - 300 MeV, would be sufficient evidence to reject the hypothesis.

The high Q-values of the reaction given in Table is 1090 μ , a range roughly twice that of the average of μ-mesons from π-decay . Measurements on the primary gave the following estimates of mass, G vs. R, 1125 ± 140 m_e. $\measuredangle$ vs. R 1125 ± 260 m_e. We will refer to this event as K_2.

To date some seven measurable examples of the decay of a heavy charged meson to one charged secondary particle have been observed by the Bristol group, and, in addition, one remarkable example has been found by the group at the École Polytechnique[23]. Before discussing the general features of these events, it will be best to summarize their characteristics in Table IV.

We may draw the following firm conclusions from the data presented in Table IV.

Supposing, first, that the particles K_1 - K_7 are identical particles, it follows from the spread in the measured energy values of the charged secondary particles, that in addition at least two neutral particles are produced by the decay of the K-particle. Further, the charged secondary particle is a μ-meson - (K_2 and K_6).

Considering those neutral particles of which the existence has been established, we may postulate the following three-body decay schemes assuming in turn that the neutral particles are, (a) two pions, (b) being due to the decay of such a particle. None was found. However, the geometry was not favourable and further, owing to the small energy of projection (1 MeV) of such a particle if it existed, the angle between the charged decay products could be little different from 180°, a circumstance which would make it extremely difficult to recognise. Armenteros et al. have described a remarkable event which, they suggest, may be interpreted in terms of a decay in flight of a charged V-particle possibly produced together with the secondary charged particle, in a two-body decay process. At first sight, it would appear reasonable to assume that the postulated two-body decay of the K_2 is the same as the cloud chamber event. However, calculations show that the Q-value of the latter must be several times that of the K_2, so that they cannot both be cases of two-body decay of the same particle.

Relation of K to Charged V-Particle.

Examples of the decay of charged V-particles first observed and interpreted by Rochester and Butler have since been found in several cloud-chamber investigations[9]. Several examples have been described by the Manchester workers among which a few were suitable for measurement.

Table IV (continued)

Particle	PRIMARY PARTICLE			SECONDARY PARTICLE						
	Length of track	Mass (m_e) $\bar{\alpha}$ vs R	Mass (m_e) G vs R	Length of track	G.D.	$\bar{\alpha}$ 100 μ	Identity	$p\beta c$	Energy	Remarks
K 6	1540 μ	1000 ±300	-	8900 μ	1.70 $g_{plateau}$	0.39± 0.051°	μ-meson from $\bar{\alpha}$ vs G.D.	59.8 MeV	34 MeV (assumed μ-meson)	Secondary is a μ-meson from measurements of grain density and scattering, and by comparison with identifiable tracks.
K 7	1300 μ	1200 +1900 -740	-	20,000 μ	$g_{plateau}$		mass < 400 m_e			

lar to those which lead to the production of π-mesons. On general grounds, however, one would expect because of their high mass, that they would be produced as an appreciable fraction of the meson component only in events of high energy. No very clear evidence of their production emerges from the work of Fowler[16] and of Camerini et al[25]. This is so partly because the events studied by those authors had a very broad distribution in energy. This work is at present being extended with improved technique by Davies et al.[25,26] at Bristol, and preliminary results of the greatest interest and importance have been announced. It would appear that as attention is directed to events of progressively higher energy, the rate of production of K-particles increases relative to that of π-mesons, and for events produced by protons of energy > 50 BeV, nearly one half of the shower particles would seem to be particles of mass $\sim$ 1250 m_e.

One of the difficulties in recognising slow K-particles in plates exposed at mountain altitudes, is the presence of an unwanted background of low-energy events such as ρ-σ decays and other two-prong events from which the K-decays must be distinguished by inspection and measurement. By making high-altitude flights with plates exposed under suitable absorbers at low geomagthe errors of observation, with the measured masses, but at the present stage, it is not possible to decide between the decay schemes.

It has been suggested by Bethe[24] that the κ and τ decay processes are but alternative transformations of the same particle, and it is of interest to examine this question in detail. If correct, the decay-scheme (a) is automatically excluded, since the hypothesis $\kappa \equiv \tau$ requires κ to be a boson. We are left with the alternative possibilities

$$\kappa \rightarrow \pi_o + \nu + \mu \qquad (b')$$

$$+ \nu + \mu \qquad (c')$$

Now, apart from a decision of the question by accumulation of a sufficient number of examples of the process in which the primary particle has characteristics favourable to direct estimate of m_κ, we may obtain evidence on this question by considering the highest value of the quantity $p\beta c$ of the secondary charged particle. Assuming $m_\kappa = 966\ m_o$, the following are <u>maximum</u> values of $p\beta c$ for the secondary μ-mesons.

micrograph is given, a particle emerging from a star appears to produce at the end of its range a small nuclear event in which two charged particles are emitted. From grain-density vs. range, the mass of the interconnecting particle would seem to be 725 m_e, while from scattering it has the higher value 1500 m_e. Owing to the possibility of fading, it might seem reasonable to suppose that the mass estimate obtained by grain-counts is too low so that it is just possible to interpret the event as a nuclear interaction by a K -particle. Two cases of particles of the same apparent mass which came to rest in the emulsion without having produced any visible interaction are mentioned, but no details of the measurements are given, but it must be remembered that secondary charged particles with grain-density near minimum would not be recorded in this emulsion. It is claimed by the authors that in the secondary nuclear event lower limits could be placed to the masses of the particles ejected from the secondary nuclear event, but it is very difficult to see how any such conclusion could be arrived at in view of the very short ranges involved A search has been made at Bristol to see if any of the primary particles of σ -stars could have been heavy mesons, but in 100 examples, no case was found in which the parent particle

V are noteworthy, and contrast strongly with those of τ-decay and the V_0^1-decay of the Manchester group[6].

It remains nevertheless a possibility that K_2 is a particle of mass and decay-scheme different from the other cases, although the grain-count and scattering measurements on K_6 would seem to provide evidence to the contrary. If there is any similarity in shape between the energy spectra of the charged secondary particles in kappa and μ-decay, it appears surprising to find in a limited sample one with the very low energy of 6 MeV, but it must be borne in mind that an event of the type K_2 is much less likely to escape observation than one of type K_1. Since the momentum and energy of the charged secondary are very accurately known from the range-energy relation, it is of interest to consider what must be the Q-value of the reaction and mass of the neutral particle, assuming two-body decay. It is found that the Q-value is very small, namely 7 MeV, and that the neutral particle must have mass 900 $\pm$ 130 m_e. All known neutral particles are thus excluded save, perhaps the V_2^0 particle of the Manchester and Pasadena workers, the existence of which is not quite certain. A search was made along the backward prolongation of the initial direction of motion of the μ-meson for an event which might be interpreted as

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One such bears a strong resemblance to K_1 and the two events may well be of the same type. This was a case of a heavily ionizing particle produced in association with a high-energy interaction in a lead plate which subsequently decayed to a single charged particle of measured momentum 150 MeV/c. It is believed that the primary particle had not quite come to rest before decaying, for it had ionization in the range 3 - 6 min. Taking the corresponding value of β as being 0.6, and assuming that the secondary is a μ-meson, application of the Lorentz transformation yields a value of the momentum in the rest system of the decaying V-particle of 100 MeV/c, a value quite consistent with K-decay. In the other cases the evidence is less certain, but it appears reasonable to suppose that at least the majority of heavy charged meson decay events are to be identified as examples of κ-decay.

Origin of Heavy Charged Mesons.

The problem of the origin of the heavy charged mesons all types of which, for brevity we will call K-particles, is of obvious importance. Their discovery in association with penetrating showers, would indicate that they are produced in circumstances simi-

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-netic latitudes, it is hoped to increase the ratio K-decays/background. This is because in such conditions the events recorded in the plates will tend to be of higher energy through the operation of the magnetic 'cut off', and plans for such flights are already well advanced.

Other outstanding problems are those of the estimation of the life-time of the particles, and the decision as to whether they have a strong interaction with nucleons. The value of the life-time is still uncertain, but there are strong reasons for believing that it is not less than 10^{-9} sec.: it is not certain that K-particles are directly produced in nucleon-nucleon collisions, or whether they result from the decay of an intermediate particle of shorter life. Information on the strength of coupling between K-particles and nucleons is meagre. There exists the case discovered by Leprince-Ringuet and LHéritier already discussed, but the nature of the particle is uncertain, and the interpretation of the event is, perhaps, open to doubt. Attention has been drawn in discussion by McCusker to work described by Wagner and Cooper. Three examples of particles of apparent mass ~ 700 m_e were found in Ilford G5 plates flown to high altitudes. In one of these, of which a photo-

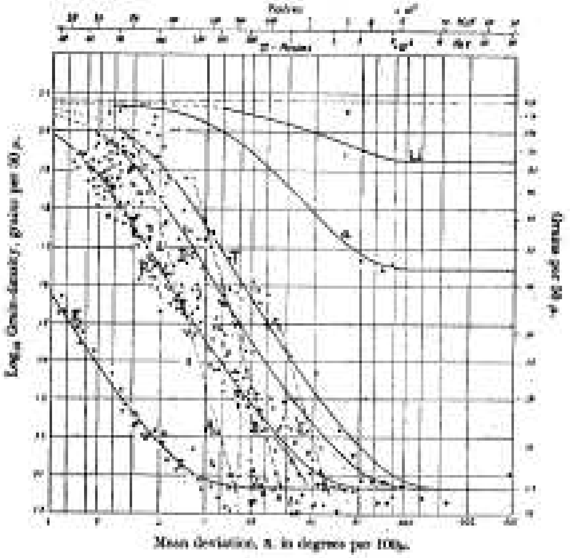

FIGURE 7

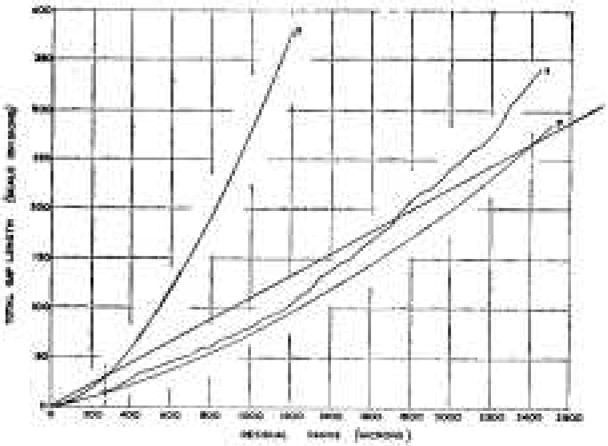

FIGURE 8

O Ceallaigh 1]

could be identified as being of mass significantly different from that of the π-meson. Investigations along these lines are still in progress.

Acknowledgments.

We wish to thank the various workers who have kindly made available to us their results prior to publication, and to thank the Proprietors of the Philosophical Magazine for permission to reproduce Plates, I, II, III and IV and Figures 6, 7 and 8 and Tables 1, II and III.

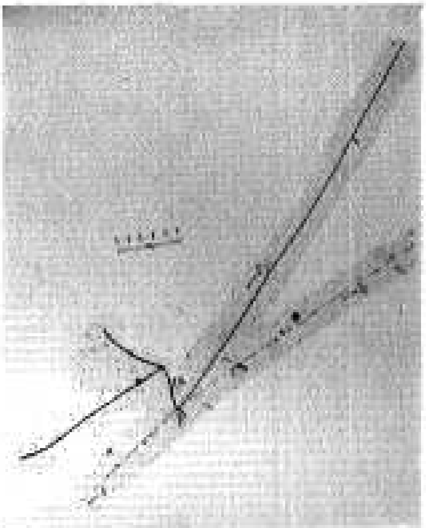

PLATE 1

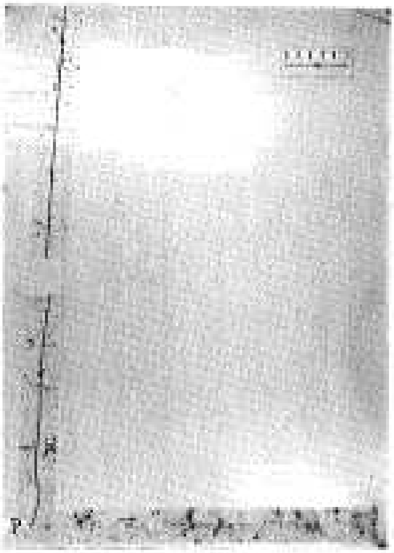

PLATE 3

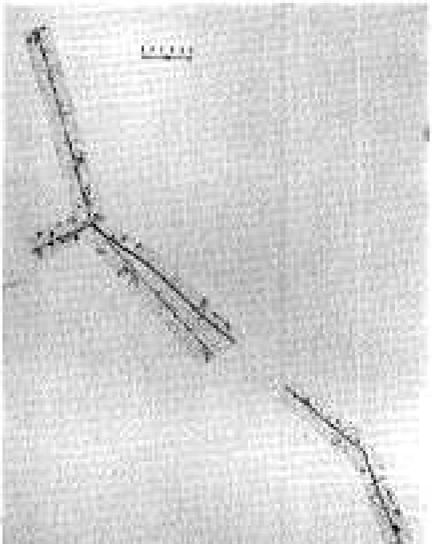

PLATE 2

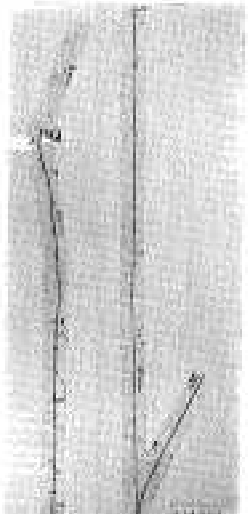

PLATE 4

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THE PROPERTIES OF MU-MESONS

E.P. George

Birkbeck College, London

1. Introduction.

About 15 years ago, absorption measurements had shown the existence of two components in the cosmic radiation: a soft component whose properties agreed with those calculated by Bethe, Heitler and others for high energy electrons, and a hard component which could not be so described, being much more penetrating.

One school of thought, supported by Blackett, suggested that the hard component consisted of electrons but that the theory broke down at very high energies, the rate of energy loss of very high energy electrons being much less than predicted by theory. The other school of thought, supported by Anderson, held that the theory was valid at all energies, and that the penetrating particles had a rest mass greater than that of the electron, since the rate of radiative energy loss is inversely proportional to the square of the rest mass. Further, they could not be protons, since protons of energy less than 600 MeV ionize suf-

-ficiently heavily to be recognisable in a cloud chamber and the number of heavily ionizing tracks found was far too small to be consistent with the identification of the hard component with protons. Hence the hard component was assumed to consist of particles of mass intermediate between that of the electron and proton, called variously, heavy electrons, barytrons, mesotrons or mesons.

The first direct evidence for the existence of such particles was obtained by Neddermeyer and Anderson in 1938, using a cloud chamber. The penetrating component was eventually shown to be mainly mesons of rest mass about 200 times the electron.

Previous to this, in 1935, a young, unknown Japanese theoretical physicist by the name of Yukawa had postulated the existence of a particle of similar rest mass in order to account for the short range forces between neutrons and protons. Yukawa modified the Laplace equation:

$$\nabla^2 \phi = 0$$

to

$$\nabla^2 \phi = \kappa^2 \phi \qquad (1)$$

a solution of which is

giving up their rest mass energy to nuclear excitation - in about 10^{-20} sec. !

A possible way out of the difficulty was proposed by Sakata and Inoue in 1943. A similar idea was suggested by Marshak and Bethe in 1947. Sakata and Inoue suggested that in the cosmic radiation there were 2 different mesons: a Yukawa particle ($Y^{\pm}$) of integral spin and a lighter meson of spin $\frac{1}{2}$, produced by spontaneous decay of the Yukawa particle:

$$Y^{\pm} \rightarrow m^{\pm} + n$$

where n is a neutral meson of negligible mass, equivalent to the neutrino. Quoting from their paper:

> "Then primary incidental protons at first create Yukawa particles by collisions with nuclei of N or O atoms existing in the atmosphere. These Yukawa particles transform into mesons instantaneously by the above process. The interaction of the latter is smaller than that of the former. Thus we observe these mesons as hard components of cosmic rays"

Furthermore, these authors showed that according to their theory, the decay of mesons occurs by the following process:

$$\begin{tikzcd}
 & Y^{\pm} + n \arrow[dr] & \\
m^{\pm} \arrow[ur] \arrow[dr] & & e^{\pm} + \nu + n \\
 & m^{\pm} + e^{\pm} + \nu + Y^{\mp} \arrow[ur] &
\end{tikzcd}$$

2. Mass.

Early measurements of the mass of the mu-meson are summarised in Table 1.

Table 1.

Method	Observer	Mass
Elastic collision.	L.R.& Gorodetsky (1941)	240 $\pm$ 20
	Hughes (1941)	180 $\pm$ 25
Momentum loss in plate	Nishina (1939)	180 $\pm$ 20
	Wilson J.G. (1939)	170 $\pm$ 20
Momentum and Ionization.	Williams & Pickup (1939)	170 $\pm$ 30
	Nielsen & Powell (1943)	210 $\pm$ 20
		180 $\pm$ 20

Much more accurate values have been obtained recently by Fretter, Brode, Franzinetti and Barkas. All of these authors have used versions of the method employing a simultaneous determination of the momentum P and residual range, R. R is determined by P x V and hence P and V may be determined separately and finally the mass. Fretter and Brode used cloud chambers, and their apparatus is shown in Fig. 3. Franzinetti used two photographic plates facing one another, shown in Fig. 4 - the so-called "magnetic sandwich". Tracks of particles are traced from one

the electron within 2 or 3 per cent, and is presumably identical with it. This is established by cloud chamber measurements of the density of droplets in tracks of mesons and electrons by Brode and by Hazen. More recently, the grain density in tracks of fast mu-mesons and electrons from electron pairs in nuclear plates exposed in our underground laboratory in London have been compared. The results were again the same within the limits of experimental error.

4. Spin.

The evidence for the spin of the mu-meson was first discussed by Christy & Kusaka (1941). These authors pointed out that since the cross-sections for the energetic collisions between mesons and electrons (knock-on electrons) and for radiative nuclear collisions of mesons (bremsstrahlung) both depended on the spin of the meson, it ought to be possible to get some indication of the spin of the mu-meson by comparing the frequency of bursts under thick absorbers with that calculated for various spin values of the mu-meson. The results of this comparison are shown in Fig. 7. From these results, these authors concluded that the spin value 1 could be definitely excluded, and that

$$\phi = g \frac{e^{-kr}}{r} \qquad (2)$$

giving a force of range $\sim 1/k$. He showed that energy should be transferred in a field satisfying (1), (2), by particles of mass m, where

$$1/k = \text{range} = \frac{\hbar}{m\,c} \qquad (3)$$

Putting $1/k \sim 1.4 \cdot 10^{-13}$ cm. gives a value for m of $\sim 200\ m_e$.

It seemed a very natural step at the time to identify the cosmic ray meson with the Yukawa particle - in any case everybody assumed them to be the same at first. As time went on however, it became clear that something was wrong - the properties of the cosmic ray meson were found to diverge more and more from those required by theory, in particular their interactions with atomic nuclei were too small by several orders of magnitude - shown up most clearly by Conversi, Pancini, Piccioni (1947). The positive cosmic ray meson decays at rest into an electron with a mean life of 2.15 microsec. By magnetic focussing Piccioni showed that negative cosmic ray mesons brought to rest in carbon showed the normal decay, whereas according to theory they should have been absorbed by the carbon nuclei -

This is the only experimental evidence that bears directly on the question of the mu-meson spin value.

Arguments of an indirect nature, based on the decay and nuclear coupling schemes of the mu-meson agree with the assignment of the value $\frac{1}{2}$ as the most probable value. These are discussed in the following sections.

5. Decay.

The fact that the mu-meson is unstable was inferred by Kulenkampf in 1938, from the so-called absorption anomaly. It has been found by several workers that the absorption in air in inclined directions was notably greater than the absorption in an equal weight of air in the vertical direction. In the inclined directions, the average density of air is less and hence the mesons have to travel a greater path length. The extra absorption was attributed to decay in the extra time taken to traverse the atmosphere. A similar consideration was advanced by Blackett (1938) to explain the observed negative temperature coefficient of the cosmic radiation. If the temperature of the atmosphere is increased, it expands, and there will be a decrease in the number of mesons surviving to sea level, owing to decay in the extra path length. From such considerations, i.e into an electron plus 2 light neutral particles. These predictions were confirmed in a remarkable way by research carried out at Bristol with photographic plates, which resulted in the discovery in 1947 of the pi-meson of mass 276. This particle , having a short life ($\sim 10^{-8}$ sec) and strong nuclear interaction, both being reasons for its rarity in the cosmic rays near sea level, seemed to behave very closely to the behaviour required by Yukawa's theory and Sakata's extension of it. Examples are shown in Figs. 1 and 2. These mesons are called pi-mesons and the secondary mesons into which they disintegrate are called mu-mesons, and it is concerning the properties of the latter that I want to confine myself. After their discovery in cosmic radiation, pi-mesons were produced by energetic nuclear collisions of artificially accelerated particles. The much greater beam intensities available from the machines resulted in a great onslaught being made on the investigation of the properties of the pi-meson, and a consequent neglect of the mu-meson. However, the mu-meson is of some interest, since it does not fit into any pidgeon-hole in the classification of fundamental particles, in the way that the pi-meson does.

the meson analogue of Fermi's β-decay theory based on the coupling scheme:

$$(P,N) \rightleftarrows (e,\nu)$$

In fact, the same Fermi coupling constant gives the lifetime of nuclear and of meson β-decay.

The value of the lifetime of the positive mu-meson was determined by Rossi and Nereson (1942) using the method of delayed coincidences. The value obtained by them was 2.15 ± 0.07 microsec. A later version of their apparatus is illustrated in Fig. 10 with which they determined the decay curve separately for positive and negative mu-mesons. Using mu$^+$ mesons arising from the decay of artificially produced pi$^+$ mesons, Alvarez (1950) determined the value 2.09 ± 0.03 microsec. The decay curve obtained by Alvarez is shown in Fig. 11.

The situation is different for negative mu-mesons. Piccioni had been unable to detect delayed electrons from mu-mesons stopped in Fe, whereas the normal number were detected in carbon. Intermediate elements were investigated by Ticho and Schein (1948) and by Valley and Rossi (1948) and the decay curves obtained by these authors for positive and for negative mu-mesons stopped in NaF and Al are shown in Figs. 12 (a) and (b).

plate to another, and their deflection in the magnetic field thus determined. The results obtained by this method are shown in Fig. 5. Barkas used essentially the same method as Franzinetti. His mu-mesons were produced by bombardment of targets in the cyclotron, and the magnetic bending was also performed by the cyclotron magnetic field. The results of this investigation are given in Fig. 6. All the results are summarised in Table 2.

Table 2. Determination of the mass of mu-mesons.

Authors:	Date	m_μ
Goldschmidt et al.	1948.	202 ± 8
Barbour.	1949	220 ± 26
Fretter et al.	1949	215 ± 2
Franzinetti.	1950	217 ± 4
Barkas et al.	1950	212 ± 6

The weighted mean gives

$$= 214 \pm 1$$

The electron mass is only known to 1 part in 1,000 so the meson physicists, at 5 parts in 1,000 have not got far to go to catch up with the best work on the electron.

3. Charge.

The charge on the mu-meson is equal to that on

The fraction of mesons decaying should be given by

$$f = \tau^{-}/\tau \qquad (7)$$

The fraction f was determined by extrapolating the decay curves to zero time, and the relation 7 verified, confirming the nuclear capture hypothesis.

Some results are collected in Table 3.

Table 3.

Element	Z	τ^-	τ^-/τ^+	f
O	8	1.89	.87 ± .08	.83 ± .08
NaF	9,11	1.23	.57 ± .06	.6 ± .06
Mg	12	0.96	.45 ± .04	.53 ± .04
Al	13	0.75	.35 ± .04	.40 ± .04
S	16	0.54	.25 ± .03	.27 ± .03

Wheeler (1949) has shown that for light elements, the capture probability $1/\tau_a$, should be proportional to Z^4,

$$1/\tau_a = \text{const} \times \sum_{\text{protons}} |\psi|^2$$

ψ is the meson wave-function.

Now

$$\psi = \left(\frac{1}{\pi a^3}\right)^{\frac{1}{2}} e^{-r/a}$$

the most likely value was 0, though the value $\frac{1}{2}$ could not be excluded.

The conclusions need revising however, for the following reasons:

(a) The theoretical curves are too high by a factor of about 3. This was pointed out by Kusaka, and arose through an overestimate of the effect of fluctuations.

(b) Not all ionization bursts at sea level are due to cascade showers arising from mu-meson knock-ons and bremsstrahlung. Only about 50 % are due to this cause, the remainder being due to nuclear disintegrations and mixed showers produced by the nucleon component of the cosmic radiation.

The question has been reinvestigated by Loeb (1951), who measured the frequency of bursts in a heavily shielded ion chamber, using electron collection. Using the method of pulse shape analysis, (Rossi et al. 1949), Loeb was able to separate out the star-type bursts due to the nucleon component. Loeb's results are compared with the various theoretical curves in Fig. 8 from which it would now appear that the most likely value of mu-meson spin is $\frac{1}{2}$.

true Z and approaches assymptotically the value 37.25 as $Z \longrightarrow \infty$

Typical values are

Z	4	8	12	16	35	56	92	∞
Z_{eff}	3.92	7.56	10.83	13.7	23	28.25	32.17	37.25

The figures in Table 3 were hardly sufficient to distinguish between a dependence on Z^4 or Z^4_{eff}. Recently, however, the capture probability has been determined for Cu(29) and Sb(51) by Harrison, Keuffel and Reynolds (1951), using scintillation counters. Their results are shown in Fig. 13, and confirm remarkably well Wheeler's Z^4_{eff} law, and provide the first experimental confirmation that the charge is spread uniformly over the nucleus.

The mu-meson capture probabilities are remarkably small, showing a very weak reaction with nuclear matter, e.g. in a large nucleus

velocity in K orbit	$\sim 10^{10}$ cm/sec.
capture time	$= 0.02$ sec.
$\therefore$ mu-meson travels	200 cm. in nuclear matter.

which corresponds to a collision cross-section of order 10^{-40} cm^2 per nucleon.

values for the lifetime of the mu-meson ranging from 1 to 3 microsecs. were obtained.

The first direct evidence of meson decay was furnished by 2 pictures obtained by Williams and Roberts (1940), using a cloud chamber in a magnetic field.

Attempts were made by various authors to determine the energy spectrum of the electrons arising from mu-meson decay - but these were not very conclusive. In 1949, the spectrum was determined by Leighton et al, using a cloud chamber in a magnetic field, and by Davies et al, using electron sensitive nuclear emulsions. The energy spectrum determined by Leighton et al, is shown in figure 9. It shows a continuous distribution characteristic of the Fermi spectrum, and shows that more than one, probably 2 light neutral particles are emitted simultaneously with the electron. The results of Davies et al, are very similar to those of Leighton et al. The measurements have been extended by O'Ceallaigh and will be described and more fully discussed by him. The decay is most readily understood in terms of pair-coupling between the (μ, μ^o) field and the electron, neutrino field: (μ, μ^o) $\rightleftarrows$ (e, ν). μ^o is a light neutral particle similar to the neutrino. In this scheme, all particles have spin $\frac{1}{2}$. This is

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6. Capture.

For the intermediate elements there is clearly a strong dependence of lifetime of the negative mu-meson on atomic number. A negative mu-meson will be captured into orbits and will quickly drop to the meson K shell, the radius of which is given by:

$$a = \frac{\hbar^2}{m Z e^2} = 2.5 \cdot 10^{-11}/Z \qquad (4)$$

so that for U, $a = 3.10^{-13}$ cm. and since $R = -.10^{-13}$ cm., the K shell is <u>inside</u> the nucleous.

For Ag, $a \sim R \sim 5.10^{-13}$ cm, and the field-strength at the K orbit is

$$E = \frac{47 \times 5 \times 10^{-10}}{(5 \times 10^{-13})^2} \times 300 = 3.10^{19} \text{ V cm}^{-1} \qquad (5)$$

The shorter lifetime of the negative mu-mesons might be explained in terms of accelerated decay produced by this intense electric field or by competition between decay and nuclear capture. Under the latter hypothesis, the apparent lifetime of negative mesons should be given by

$$\frac{1}{\tau^-} = \frac{1}{\tau} + \frac{1}{\tau_n} \qquad (6)$$

where τ is the lifetime for spontaneous decay and τ_n is the lifetime for nuclear capture.

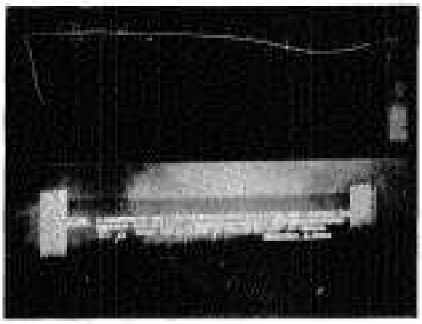


FIGURE 1—A positive π- meson decays at rest into a μ- meson which in turn comes to rest and decays into an electron.

FIGURE 2—A negative π- meson is captured by a nucleus in the emulsion and gives rise to a disintegration "star" of three branches.

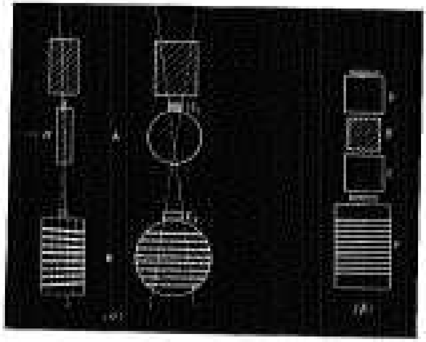


FIGURE 3—The cloud chamber arrangements of Fretter (a) and Brode (b).

George]

and if $a \gg R$,

$$\frac{1}{\tau_a} = \text{const} \times Z \times \frac{1}{a^3} = \text{const} \times Z^4 \qquad (8)$$

which may be written

$$\frac{1}{\tau_a} = \frac{1}{\tau} \left(\frac{Z}{Z_0}\right)^4 \qquad (9)$$

where τ is the decay lifetime, and Z_0 is the transition element for which $\tau_a = \tau$. Experimentally, Z_0 is found to be about 10.

Since, for large nuclei the K-orbit lies inside the nucleus, the hydrogen-like approximation is no longer valid.

Wheeler has determined the wave-function for mu-mesons moving in a potential

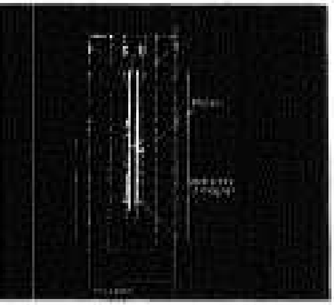
$$V(r) = - \left(\frac{Z e^2}{R}\right) (1.5 - 0.5\, r^2/R^2) \qquad r < R$$

$$= - \left(\frac{Z e^2}{r}\right) \qquad \text{for} \quad r > R$$

corresponding to the charge being uniformly spread over the nucleus. Equation (9) becomes replaced by

$$\frac{1}{\tau_a} = \frac{1}{\tau} (Z_{eff}/Z_0)^4$$

Z_{eff} is the effective Z, which is smaller than the



...re 4—The " magnetic sandwich " arrange-
... of nuclear emulsions in a magnetic field
used by Franzinetti.

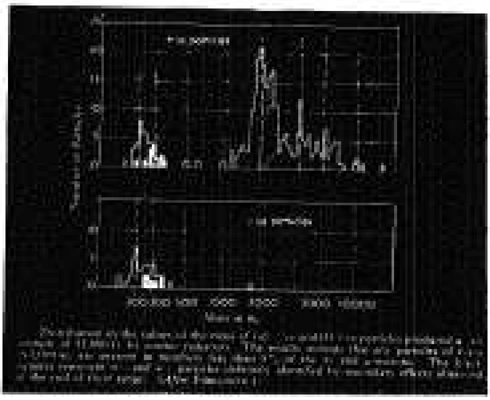


FIGURE 5

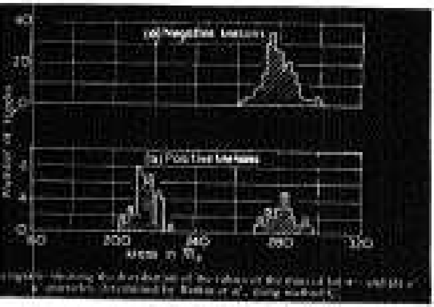


FIGURE 6

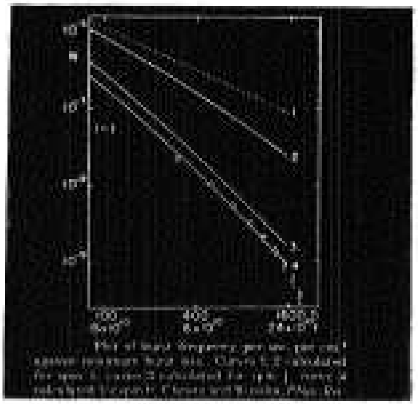


FIGURE 7

Such a small cross-section has been interpreted by Tiomno and Wheeler as indicating a reaction of the type

$$\mu^- + P \rightarrow N + \mu^0$$
$$(\mu^-, \mu^0) \rightleftarrows (P, N) \qquad (10)$$

where μ^0 is a light neutral particle, paired with the charged mu-meson in a formal way, as in the β -decay scheme. Again, such a coupling fits most simply with the assignation of spin values $\frac{1}{2}$ to the μ^- and μ^0 particles.

Reaction (10) in free space should lead to a neutron recoil energy of 5 MeV. Due to the internal motion of the protons inside a nucleus however, there will be a spread in recoil energies, and the average is about 15 MeV, sufficient to make a small disintegration. Because of the Coulomb barrier neutrons will be preferentially emitted.

The emission of neutrons following mu-meson capture has been observed by Groetzinger and McClure who found 1.96 $\pm$ .72 neutrons per negative mu-meson in lead, and by Sard who found 2.16 $\pm$ .15 in lead.

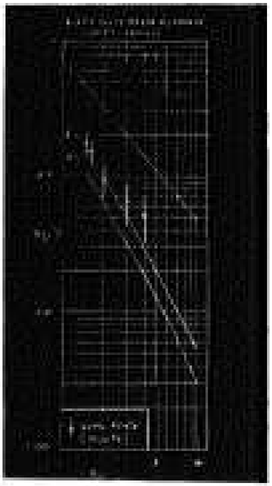


FIGURE 8—Comparison of results of Loeb, obtained with a fast ion chamber, with theoretical values for various values of μ- meson spin.

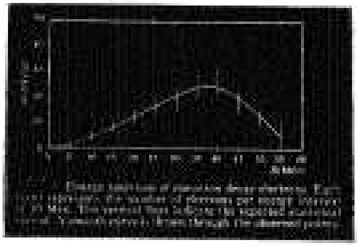


FIGURE 9

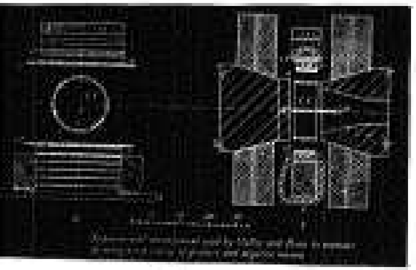


FIGURE 10

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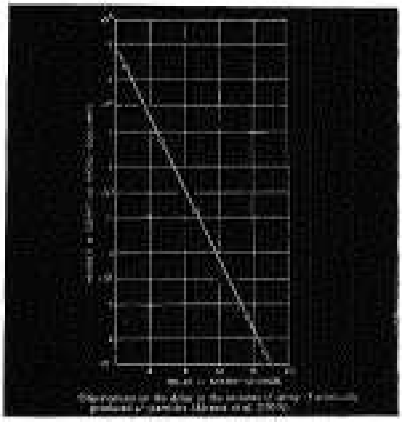


Figure 11

Figure 12a

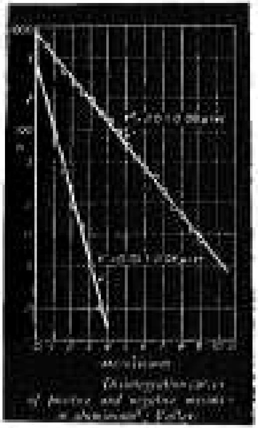


Figure 12b

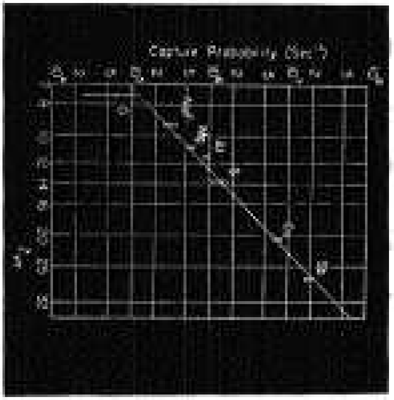


Figure 13

Plot of capture probability against Z_{eff} on a logarithmic scale. The straight line is drawn with a slope equal to 4.

George]

THE INTERACTION OF CHARGED COSMIC RAYS WITH URANIUM NUCLEI

W. Galbraith and W.J. Whitehouse
Atomic Energy Research Establishment, Harwell.

Introduction.

The experiment to be described was designed originally to determine whether the capture of a negative μ-meson by a uranium nucleus led to fission of that nucleus.

Dr. George in a preceding paper has discussed the nuclear interaction of the negative μ-meson and made reference to a paper by Wheeler on this subject. In Wheeler's paper it is pointed out that in the case of uranium, fission of the nucleus may result from capture of the μ-meson in one of two possible ways.

(i) The energy released in the 2S-1S meson orbit transition may be sufficient, even when allowance has been made for the critical shape of the nucleus just prior to fission, to cause fission of the nucleus by a photo-process.

(ii) The ultimate capture of the μ-meson visualised in terms of the charge-exchange reaction

coincidences between charged cosmic rays which pass through a G.M. telescope of two trays A and B, and which fail to discharge an anti-coincidence tray D, and fission pulses which are recorded in an ionization chamber C placed immediately below B and above D. (See Figs. 1 and 2.) Such a coincidence event we shall call A B C-D.

The amount of uranium which can be put in the chamber is limited. The uranium is deposited in the form of thin layers of oxide, and the maximum practical thickness of such a layer to detect fission fragments is about 2 mgm/cm^2. For any thicker films, the absorption of fission fragments in the film becomes serious and no gain in fission pulse counting rate would be obtained. In order to work with gram quantities of uranium, the chamber was designed to accommodate 4000 cm^2 of uranium film of 2 mgm per cm^2 thickness.

The rate of spontaneous fission pulses observed was about 120 per hour and the spontaneous fission rate for natural uranium is approximately 25 fission per hour per gram (Segre LADC 975). Thus the effective mass of uranium in the chamber is 5 gms $\pm$ about 3 %.

Estimates of the rates of other possible ABC-D events have been made and, with one exception, the frequency of occurrence of such events is smaller than the background of accidental coincidences.

The exception is the expected rate due to fast proton interaction with the uranium nuclei. We obtain some idea of the rate to be expected, from a knowledge of the proton component at sea-level, if we assume that the cross-section for interaction is the geometrical cross-section of the uranium nucleus. Protons with momenta in excess of 0.76×10^9 ev/c will penetrate the 3" lead, and the flux of protons at sea level with momenta greater than this is about 0.55×10^{-4} per sq.cm. per sec. (Mylroi and Wilson, Proc.Phys.Soc. A 64, 404, 1951). A rough calculation of events due to fast protons then gives 7 events per 1000 hours.

An alternative method of obtaining an estimate of the number of 'stars' or fissions due to protons is based on the number of stars with charged primaries produced per c.c. per day in photographic emulsion at mountain altitudes. This rate is reduced to a sea-level value and we assume that the percentage of stars with charged primaries remains unaltered. From this we calculate that the number of fissions to be expected

efficient anti-coincidence tray owing to the necessity of supports for the chamber, but its presence was essential to reduce the background rate of accidental coincidence pulses below the expected rate of ABC-D coincidences.

The H.T. for the counters was supplied by batteries and the output pulses from the trays were fed into amplifiers which sharpened the rising edges of the pulses before feeding them into the coincidence unit.

(3) The coincidence unit.

The outputs of the various channels of this unit were easily recorded, (see figure 1) and in practice daily measurements were made of the rates A, B, D, AB, ABD and AB-D. The rates C, ABC, ABC-D and AB-D were continuously recorded during a run. The operation of the whole apparatus was checked each day by feeding in test pulses at a known fast rate into the input grid of the C head amplifier. By noting that the resulting rate of ABC and ABC-D events corresponded to the expected accidental coincidence rate, this check also showed that the resolving time of the coincidence unit was not varying.

(4) Pulse recording.

The ABC pulse was used to trigger an oscilloscope and camera, and a photograph of the fission pulses was

$P + \mu^- \rightarrow N + \nu$, may lead to an excitation of the nucleus favourable to fission, rather than to neutron emission. (The emission of neutrons following μ^- meson capture in lead is now well established and understood in terms of the excitation of the nucleus following the charge exchange reaction).

In the case of uranium process (ii) is the more probable one leading to fission, but Wheeler points out that the rate of μ-meson-induced fission at sea-level would be small compared with the spontaneous fission rate of uranium.

Tat cosmic rays at sea-level do not contribute appreciably to the spontaneous fission rate was first shown by the early work of Flerov and Petrzhak (1940). They observed spontaneous fission pulses from films of uranium oxide in an ionization chamber at ground level and obtained no change (within statistics) in the counting rate when the chamber was taken below ground into the Moscow Metro.

It is evident that some form of coincidence experiment is necessary in order to detect fissions following the stopping of charged cosmic rays by uranium.

Present Experiment: Estimated Rates.

The experiment consists essentially of recording

Table 1. (continued)

Under 3ft. Concrete						
Total hours	Obs. ABCD	Expect accid. ABCD	Obs. ABC-D	Exp. accid. ABC-D	Expected ABC-D rate due to μ^-	p
Run (a) 1340	10	$\simeq 6$	5	$\simeq 2$	$\simeq 37$	$\simeq 2$

At the end of run (a) an investigation of the ABCD rate led us to believe that it might be instrumental because a source of interference was detected in the C amplifier noise level from the AB channel. This was eliminated by careful earthing and avoidance of earth loops. It was also thought that it might be due to a shower event which was dense enough to similate a fission pulse in the ionization chamber.

The latter explanation was shown not to be the case by including an additional small tray of 8 counters, coupled alternately, immediately below the chamber (see fig. 8) and looking for a simultaneous discharge of a pair of these counters in coincidence with an ABCD event. In a further run Table 1 (b) 6 ABCD events were recorded, none of which were in coincidence with

A calculation of the expected rate of μ-meson absorption in this mass of uranium is based on the experimental data of König (Phys.Rev. 69, 590, 1946). He found that 30.6 gm/cm^2 of lead was effective in stopping 1 ‰ of the meson beam passing through 1 cm^2, and, if we assume a similar absorption takes place in uranium, the mean rate of μ$^-$mesons stopping in the uranium is approximately 0.035 per hour. This will be the rate of A B C-D coincidences on the assumption that each stopped μ$^-$meson produces fission. In this estimation, the value of the meson flux through the chamber is that measured by the G.M. telescope (0.75 particles per sq.cm. per minute).

The average random accidental rate ABC-D, calculated from the measured AB-D rate (350 per minute) the C rate (120 per hour) and the resolving time (1 μ sec.) is approximately 0.0014 per hour.

There are other possible sources of events ABC-D, the most important being due to the emission of γ rays at the instant of fission of uranium nuclei, which could be simultaneously detected in trays A and B, and not in D. The efficiency of detection of A and B to these γ rays was measured, and 3" of lead were inserted between A and B. In this way the γ detection efficiency of A was made negligibly small.

In the laboratory, 5 ABC-D events were recorded in 1340 hours, during which there were, in all, 15 ABC events. The expected accidental rates would be 2 ABC-D and 10 ABC during this time.

It will be seen that, under the concrete, the ABC-D rate has been significantly reduced, whereas if the events in the trailer were to be attributed to μ^- mesons we should not expect any measurable change under the concrete.

There is no measurable ABCD rate above accidental background under the concrete.

Conclusions:

The results of the experiment give no positive evidence for the interaction of the μ^- meson, leading to fission, with uranium nuclei. The statistics are so poor, however, that the results are consistent with an upper limit of 0.3 for the probability of a captured μ^- meson giving rise to fission. The results would still be consistent if we attributed the observed rate under concrete to protons.

It may be that the charge-exchange reaction between the μ^- meson and the uranium nucleus leads to neutron emission in a large proportion of cases, but a search for these neutrons in an experiment along the in the uranium films is about 8 in 1000 hours. This agrees satisfactorily with the previous considerations.

Apparatus:

(1) Ionization chamber and associated amplifiers.

Electron collection was essential in order to work with a resolving time of 1 μ sec in the coincidence circuits. Consequently the gas filling was argon at a pressure slightly above atmospheric and it was continuously purified by convection flow over hot calcium. (The general arrangements of the purifying system will be seen in Figure 3). The head and main amplifier were of conventional design, with paper dielectric condensers replacing all electrolytics. This modification lowered the spurious pulse background of the amplifier.

(2) G.M. Trays.

A consisted of 19 brass counters giving an effective area 4900 cm^2 and B consisted of 15 glass counters of area approximately 2900 cm^2. D comprised 19 brass counters, together with 3 banks of 3 counters each, around the accessible sides of the chamber. In each tray the counters were connected in parallel with one another. It was not possible to make D a highly beam of known composition from a cyclotron, and look for a meson track ending in an asymmetrical fission track. Such an experiment would require very careful control experiments especially in regard to a further investigation of the fission track characteristics of π^- induced fissions, in order to obtain any quantative results for the μ^- mesons.

Fig. 1. Block diagram of apparatus for CR-Fission experiment.

Fig. 2. Photograph showing arrangement of G M. trays and fission chamber inside the trailer. Shower tray is seen below the fission chamber.

Fig. 3. Photograph showing other view of apparatus, including purifier system on ionization chamber.

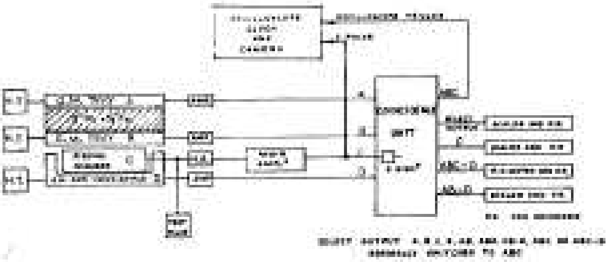
taken. Simultaneously a clock was illuminated so that the time was recorded. Examination of the pen recorder charts together with the oscilloscope camera record, showed that the pulse obtained was genuine and not spurious.

The whole apparatus was operated from a separate motor generator supply and so was independent of mains surges.

Results: The apparatus was first set up in a trailer with a thin metal roof. The results of two runs are given in Table 1.

Table 1: Observations with Uranium Foils

Trailer						
Total Hours	Obs. ABCD	Expect Accid. ABCD	Obs. ABC-D	Exp. Accid. ABC-D	Expected ABC-D rate due to μ^-	p
Run (a) 730	18	$\simeq 6$	11	$\simeq 2$	$\simeq 23$	$\simeq 6$
Run (b) 480	8	$\simeq 4$	6	$\simeq 1$	$\simeq 15$	$\simeq 4$



BLOCK DIAGRAM OF APPARATUS FOR C.R.-FISSION EXPERIMENT

FIGURE 1

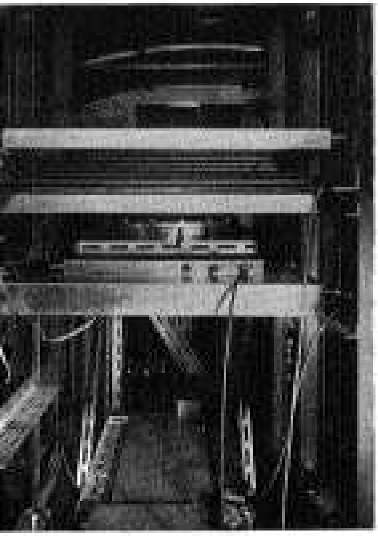


FIGURE 2

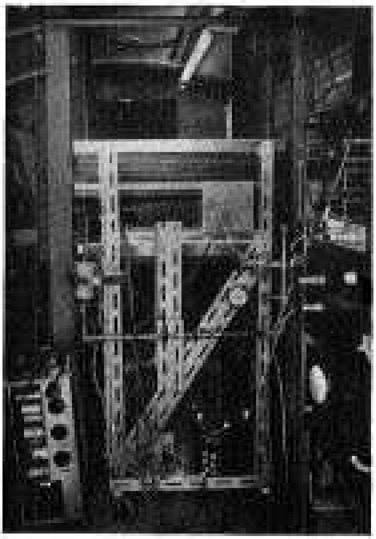


FIGURE 3

eath & *Whitehouse*]

discharges in the shower tray.

When we consider the instrumental effect we are satisfied that on the average no more than one ABC-D pulse out of the 11 observed in run(a) might have been due to this effect and so we feel justified in including the results of this first run in Table 1.

The background rate of accidental coincidence was measured by removing the chamber from under the B tray and placing it about 2 metres away from the G.M. trays. In such background runs totalling 850 hours 8 ABC and 2 ABC-D coincidences were recorded. These rates agree with the calculated rates of 5 and 2 respectively.

As a further control experiment, the uranium films were replaced with thin (0.002") lead foils, and with the chamber under the B tray once more, no ABC-D events were recorded in 470 hours.

It was evident that the ABC-D rate might, as previously discussed, be explained, in part, by the interaction of the sea-level proton component with the uranium and since it was impracticable to add any absorber above the apparatus in the trailer the apparatus was set up again in a laboratory with a concrete roof 3ft. thick. Such a thickness of concrete was expected to attenuate the proton component by a factor of 4-5.

THE INTERPRETATION OF THE INTENSITY-DEPTH CURVE FOR THE COSMIC RADIATION

J.C. Barton

(Birkbeck College, University of London)

Introduction

A considerable number of observations have been made of the intensity of the cosmic radiation at various depths below the surface of the earth. Several authors have analysed these results in order to obtain information on the interaction of cosmic ray particles of very high energy. The purpose of the present paper is to review the work which has been done recently in this direction, to carry out calculations based on more recent data and to indicate why different authors have arrived at different conclusions.

The Experimental Results.

Most of the earlier results were obtained with ionization chambers and, because of the effects of local radioactivity, are no longer considered satisfactory. The various measurements which have been made using counter systems have been summarised by Evans (1951) and, after normalization, plotted on a lines of Sard et al (Phys.Rev. 76, 1134) would be rendered impossible by the relatively high background rate of neutrons emitted in spontaneous fission. It is not possible to distinguish by such an experiment those neutrons emitted by a nucleus following μ^-meson capture and those emitted in fission.

The cloud chamber has the same disadvantage as the ionization chamber, when we consider the limitation to the amount of uranium that can be incorporated.

There is an experiment which might be possible using photographic plates. The Berkeley group (UCRL 1201) has investigated π^-meson induced fission in uranium and it is found that, within the statistics, capture of a π^-meson by a uranium nucleus leads in every case to symmetrical fission of the nucleus i.e. the tracks of the two fragments which end in the emulsion are of the same length. μ^-meson capture on the other hand does not lead to a highly excited nucleus, and if fission did follow from capture of some of the μ^-mesons, we should expect asymmetrical splitting of the nucleus. Here the lengths of tracks would be roughly in the ratio 1.4:1. The experiment would be then, to expose uranium soaked plates to a meson down to a minimum of about 10% at a few metres below the surface and then slowly increases to about 30% at 1000 m.w.e. It is seen that neither correction is very important compared with the decrease in the vertical intensity by a factor of one hundred thousand. The final curve may be normalized to the known absolute vertical intensity of the hard component at sea level.

Direct Comparison with Differential Spectra.

By differentiating the corrected and normalized intensity-depth curve it is possible to obtain the absolute differential range spectrum for the hard component. This is shown as the full line in Figure 2.

The differential spectrum of μ-mesons has been determined by George and Evans (1950) by measuring the number which come to rest in photographic emulsions exposed at different depths underground. These measurements have been extended by Creamer (1951), who has also determined the corrections necessary to obtain the absolute differential intensity. It is seen from Figure 2 that the two sets of measurements are in excellent agreement, which shows that at least the large majority of the particles at those depths must be μ-mesons.

The differential momentum spectrum at sea level has been measured by Caro et al. (1951). These

of the mesons at their point of production. The first stage of the calculation is then to find the momentum spectrum of mesons at sea level, which can then be converted to an integral range spectrum. The main difficulty in the calculation is that the π-meson may interact with one of the nuclei in the air before it can decay into a μ-meson.

The assumptions which have to be made are as follows:

1) The atmosphere is 'exponential', of thickness 1,000 gm/cm^2 and the 'height of the homogeneous atmosphere' is 8 Km.

2) All processes are collimated in direction: this should be true at such high energies.

3) The primaries cascade in a manner similar to that given by Heitler and Jánossy (1949) so that their energy spectrum is the same throughout the atmosphere. The attenuation length is taken to be 125 gm/cm^2, but calculations for a length of 30 gm/cm^2, corresponding to the case of heavy primaries, have shown that the results are quite insensitive to the exact value used.

4) The rest lifetime of the π-meson is 2.5 x 10^{-8} seconds and that of the μ-meson is 2.15 x 10^{-6} seconds.

(1951), might make a considerable difference to the results, but their existence has not yet been established.

It is convenient to divide the momentum range into two regions. For momenta below about 10 GeV/c the lifetime of the π-mesons is so short that the probability of their interacting before they decay into μ-mesons can be neglected. Then the intensity of μ-mesons with momentum P at sea level is given by

$$N(P)dP = \frac{1}{0.8} \int_{z=0}^{1000} \left\{ e^{-\frac{z}{125}} \frac{dz}{125} \right\} \frac{A\,dP}{\left\{ \frac{1}{0.8} \left[P + 2.2\left(1 - \frac{z}{1000}\right) + 0.6 \right] \right\}^{3}}$$

$$\times \left\{ \frac{P \quad 2.2\left(1 - \frac{z}{1000}\right)}{P \times \frac{z}{1000}} \right\}^{-\frac{Hm\mu}{T\mu\,(P + 2.2)}}$$

where z is the depth in grams below the top of the atmosphere. The three terms of the integrand are, respectively, the number of primary interactions, the differential meson intensity at that point and the probability of a meson produced at z reaching sea level without decaying; the third term is given, for example, in Jánossy (1948).

For momenta greater than 10 GeV/c the probability of the u-meson decaying is very small and it is also single graph, which is reproduced here as Figure 1. (Note that all the depths are increased by 10 metres so that sea level may be included on a logarithmic plot). The experimental conditions were particularly satisfactory for the measurements of V.C. Wilson (1938) down to a depth of 1100 m.w.e. In this region the other results seem to be in reasonably good agreement with those of Wilson and may be regarded as confirming them. Wilson's point at 1400 m.w.e. was obtained by inclining the telescope, which does not necessarily give an equivalent result, as will be explained later. Unfortunately there has not yet been any independent confirmation of Miyazaki's important measurement at 3000 m.w.e.

Measurements with a counter telescope are straightforward and only two small corrections are necessary. Firstly, the angular distribution of the radiation becomes more strongly collimated at greater depths; it changes from a $\cos^2 \theta$ distribution near the surface to a $\cos^3 \theta$ one at about 1700 m.w.e. This effect has to be taken into account when converting readings obtained with a counter telescope of finite solid angle to the absolute vertical intensity. Secondly the proportion of soft radiation in equilibrium with the hard component depends on the energy of the particles; this proportion decreases from sea level is the same as that of the primaries, the resulting partial differential equation can be integrated directly. For other values of the interaction length the method breaks down and, although it might be possible to solve the equation by means of some type of integral transform, the method given above seems more direct.

The expressions given have been evaluated for various values of P and the resulting differential momenta spectra have been plotted in Figure 3. Two spectra are shown for the μ-mesons, the upper being the case when the π-mesons do not interact and the lower one when they have a nuclear geometric interaction. In the former case there is also an appreciable number of π-mesons at the higher momenta; these would not be distinguished from the μ-mesons so the total intensity is also plotted. The dotted line shows the effect of ignoring the decay of the μ-mesons. The experimental points of Caro et al. (1951) are seen to lie on the curves but it is not possible to distinguish between the two possibilities for the π-meson interaction.

Conversion to Integral Range Spectra.

This final stage of the calculation requires an additional assumption concerning the rate of energy results are relative values and have been multiplied by an arbitrary factor to give the dotted line of Figure 2. In converting from a momentum spectrum to a range spectrum it has been assumed that the energy loss of mesons underground was 2.2 MeV/gm. The agreement obtained is reasonable, but the measurements are not sufficiently precise to show whether the assumed energy loss is correct. Accurate measurements of the spectrum at higher moments will be particularly useful for resolving this question.

Theoretical Analysis.

In this section a method of calculating the intensity curve will be outlined and the necessary assumptions listed. It will be shown subsequently that the numerical results obtained do not conflict with those obtained by previous workers.

The basic process which the analysis has to describe is that of a primary cosmic ray particle interacting in the atmosphere to produce a π-meson, which then decays into a μ-meson. In principle it would be possible to start the calculation from the energy spectrum of the cosmic ray primaries but at present this method is unsatisfactory, because there is no established theory of meson production. Instead it seems better to assume directly the momentum spectrum

Bremsstrahlung	1.0×10^{-6}	
Pair Production	1.6×10^{-6}	
Nuclear Events	0.5×10^{-6}	
Penetrating Pairs	2.0×10^{-6}	or, theoretically, 3.0×10^{-6} (Garelli and Wataghin).

The final expression for the energy loss is therefore

$$dE/dx = -(2.2 + 5.0 \times 10^{-6}\ E)\ \text{MeV/gm, with } E \text{ in MeV.}$$

Since there may well be other effects at very high energies the second term in this expression is likely, if anything, to be an underestimate. The expression is integrated to give the dependence of range on energy.

The momenta spectra have been converted into range spectra with and without allowing for the additional energy loss. After integration the results have been plotted in Figure 4, which also shows the corrected experimental points.

Discussion of the Theoretical Results.

At first sight Figure 4 appears to give strong evidence for the assumption that π-mesons of high energy do not interact strongly with nuclei. The same conclusion was reached by Garelli and Wataghin (1950) using a slightly different method of calculation. Nevertheless there is considerable doubt as to the validity of this conclusion. The shape of the calculated curve depends critically on the form of the momentum spectrum at the

5) The momentum of the μ-meson is always 0.8 of the momentum of the π-meson producing it.

6) The momentum spectrum of the π-mesons at the point of production is given by

$$dN = \frac{A}{(p + 0.6)^3} \, dP \qquad P \text{ in GeV/c}$$

This agrees with the measurements of Camerini et al. (1950) for momenta up to about 1 GeV/c, but its extension to much higher momenta is an assumption which will be reconsidered later.

7) Either the π-mesons are not absorbed or else they have a catastrophic absorption length of 62 gm/cm^2. These represent the two extreme possibilities; the latter one is known to be true at low momenta but the Heitler theory of mesons suggests that the cross section falls off inversely with the momentum.

8) The effects due to the presence of other types of meson are ignored, since the lifetimes of the K-mesons and V-particles are thought to be very short. It may be noted also that 6) refers to the spectrum of the π-mesons and is independent of any short intermediate link. The frequent occurrence of stable neutral K-mesons, as suggested by Hayakawa

production and at sea level, but it will be difficult to extend these measurements to momenta much above 10^{11} eV/c.

Additional Evidence on the Interaction of π-Mesons.

In view of the above difficulties in interpreting the intensity depth curve it is worth considering other ways of obtaining information on the π-meson interaction. Three methods have been suggested:

1) If the π-mesons do not interact strongly there should be an appreciable number of them underground, as can be seen in Figure 3. It would however be difficult to distinguish them experimentally, particularly because of the background of locally produced π-mesons.

2) Duperier (1949) has shown that the interaction of π-mesons leads to a correlation between meson intensity and upper air temperature. Macanuff (1951), working at a depth of 60 m.w.e. has obtained a similar effect which should throw light on the interaction of π-mesons with an average energy of about 2×10^{10} eV. Unfortunately the data on the temperature distribution and variation in the upper atmosphere is not sufficiently complete to allow any definite conclusions to

possible to assume that $z=0$ in the second term of the above integral without introducing any appreciable error. On the other hand the possibility of the π-meson interacting must now be considered. If the π-meson is produced at a depth z and decays at a depth z', we have to calculate the compound probability of the π-meson neither interacting nor decaying between z and z', and then decaying in dz'. In this way the intensity of μ-mesons at sea level is found to be

$$N(P)\,dP = \frac{1}{0.8}\,\frac{A\,dP}{\left[\frac{1}{0.8}\,(P+2.2+0.6)\right]^{3}}$$

$$\times \int_{z=0}^{1000}\int_{z'=z}^{1000} e^{-\frac{z}{125}}\,\frac{dz}{125}\; e^{-\frac{z'-z}{L_\pi}}\left(\frac{z'}{z}\right)^{-n}\frac{n\,dz'}{z'}$$

where $n = \dfrac{H\,m_\pi}{T_\pi\,(P+2.2)} \doteq \dfrac{200}{P}$; L_π = interaction length of π-mesons.

A rather similar expression can be used for the intensity of π-mesons at sea level.

A alternative method of calculation, which was first used by Greisen (1949), is to set up a diffusion equation for the π-mesons in the atmosphere. Provided that the interaction length of the π-mesons accurate measurement of the intensity depth curve should enable this to be done.

Acknowledgment.

I am indebted to Dr. E.P. George for many valuable discussions during the course of this work.

loss of fast mesons. The expression

$$dE/dx = - 2.2 \text{ MeV/gm}$$

is believed to be correct for momenta up to about 10 GeV/c, although there is some doubt as to the exact numerical value. More important is the question of additional energy losses at higher momenta. It is approximately true that these losses increase linearly with the energy so that the modified expression for the energy loss is $dE/dx = - 2.2 - b\,E$. The additional term consists of several components, which are either given theoretically or else may be determined experimentally. The effects due to Bremsstrahlung and pair production follow directly from quantum electrodynamics and have been calculated by Hayakawa and Tomonaga (1949). Rather less certain are the nuclear effects due to the Coulomb charge of the meson; the theoretical estimate of Gerelli and Wataghin (1950), based on the theory of Sneddon and Touschek (1949), is considerably greater than that deduced experimentally by George (1951) from studies of the nuclear interactions. George has pointed out that the presence of penetrating pairs underground represents a considerable energy loss and this may explain the discrepancy. The various components of 'b' are thus:-

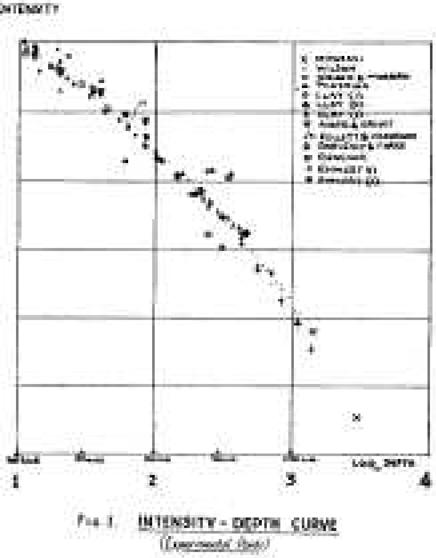


Figure 1—Intensity-Depth-Curve. Experimental Points.

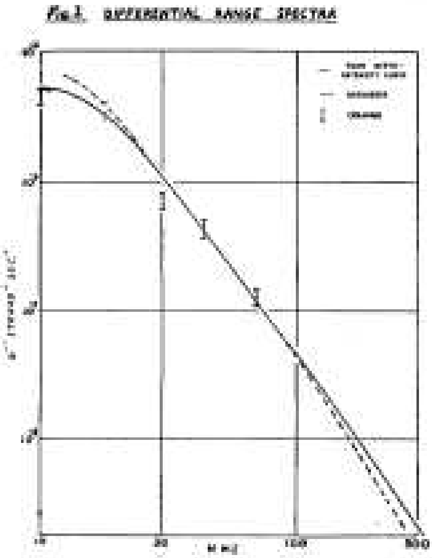


Figure 2—Differential Range Spectra.

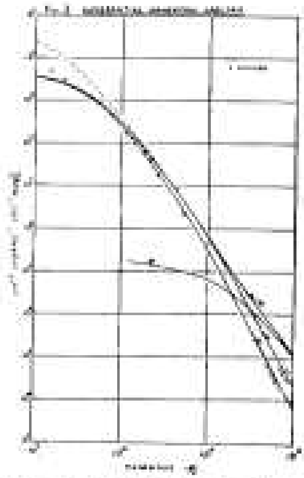


Figure 3—Differential Momentum Spectra.

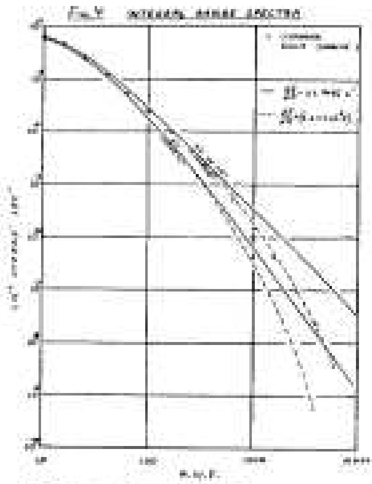


Figure 4—Integral Range Spectra.

point of production (assumption 6), which rests either on extrapolation from an entirely different energy region or else, in the case of Gerelli and Wataghin, on a particular theory of meson production for which there is no experimental confirmation. The extent of the dependence on this assumption can be seen from the fact that changing the exponent in the momentum spectrum from -3.0 to -2.7 brings the curve for strongly interacting π-mesons into agreement with the experimental results. The possibility that the increasing slope of the intensity/depth could be explained in terms of the π-meson interaction was first pointed out by Greisen (1948). The latter author ignored the additional energy loss of the fast μ-mesons. Hayakawa and Tomonaga (1949) reached the conclusion that the π-meson interaction was finite but less than geometric; they used an early value for the lifetime of the π-meson and probably underestimated the term for the additional energy loss.

Clearly it is not possible to set down any definite conclusions at the present time. Indeed it is unreasonable to expect to be able to deduce from one set of experimental data information on three distinct processes: the production spectrum, the π-meson interaction and the μ-meson energy loss. Ideally, direct measurements could be made of the momentum spectrum at the point of

A LIQUID SCINTILLATION DETECTOR OF LARGE AREA FOR EXPERIMENTS ON DELAYED PARTICLES IN EXTENSIVE AIR SHOWERS

J.V. Jelley
Atomic Energy Research Establishment, Harwell

Introduction.

It was early realized that, in a study of delayed particles in the extensive air showers, a liquid scintillation counter would offer definite advantages over the use of Geiger counter tubes as hitherto employed. There are three main advantages:- (i) High speed and short delay. With the scintillation detector it is possible to measure time delays down to $\sim 10^{-8}$ sec., with confidence that delayed spurious pulses do not arise in the scintillation medium. (ii) Avoidance of detector paralysis:- With Geiger counter trays there is always an uncertainty, in experiments of this nature, as to the effective area of the tray for observing delayed events, since an unknown number of counters will have been discharged by the prompt electrons. With the scintillator there is virtually no paralysis, and the rapid recovery is be drawn.

3) Hayakawa (1951) has pointed out that the problem could be settled by a measurement of the angular distribution at a large enough depth underground. Suppose the vertical intensity underground is given by $I(z,0) = Az^{-y}$, then it is customary to write

$$I(z,\theta) = I(z.\sec\theta,0) = Az^{-y}\cos^{y}\theta = I(z,0)\cos^{y}\theta$$

This argument breaks down if the π-mesons interact, as it is then no longer true that $I(z,\theta) = I(z.\sec\theta,0)$: this difference occurs because the path of the π-meson lies in a less dense part of the atmosphere for the inclined direction. Hence the angular distribution is less sharply collimated if the π-mesons interact. For a sufficiently high energy it is found that the exponent of the angular distribution should be one less than the exponent of the intensity depth curve.

Only one set of observations has been made at an adequate depth. Bollinger (1951) has found that the exponent of the angular distribution at 1,700 m.w.e. is 3.04 ± 0.13, and that of the intensity depth curve 3.50 ± 0.30. The results are thus not sufficiently accurate to determine this question, though a more of an E.M.I. 11-stage photomultiplier tube type 5311 operated at 2 KV. The large auxiliary vessel shown in figure 1 serves to take up the considerable thermal expansion of the 15 litres of solution and to provide a small head of pressure on the gasket, preventing air bubbles from creeping in.

There are two output channels from the head amplifier: (i) a slow channel, capable of pulse rise-times of the order of 1 μ-sec., feeds a discriminator and one channel of a triple coincidence unit, the other two channels of which are fed from Geiger counter trays each of area 560 cm^2. These and the scintillation detector are situated on a flat roof 100 feet from the rest of the apparatus and are arranged at the apexes of a equilateral triangle of sides 5 metres. The output of the coincidence unit is used to fire a triggered recording oscilloscope. (ii) A fast channel, which feeds the deflection plates of the oscilloscope via a delay cable and a low gain amplifier of the distributed-line type, with a response up to 100 Mc/sec.

Measurements of Efficiency.

The efficiency of the detector was measured in a triple coincidence arrangement shown in figure 2(c) in which the incident cosmic-ray particles at sea-

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Finally, the efficiency of the detector at the edges of the container, for vertically incident particles, was measured, by replacing the trays A and C by two small single counters. Values for the efficiency, obtained in the same way as before, were (91 ± 6) % and (91 ± 7) % at the two bias levels of 12.5 and 5 volts respectively, corresponding to single rates in the detector of 1,100/min. and 2,400/min.

Results of Preliminary Experiments.

A brief summary follows describing results of some preliminary experiments carried out in search of delayed particles in the time-delay region of 0.5 - 40 μ-seconds. A detector similar to the above was used but of smaller dimensions (400 cm^2 area and depth 9 cms) in conjunction with the air-shower selector described above. On 5057 recordings, carried out over 458 hours, there were observed 11 delayed particles on the time base. The time-base duration was sometimes changed between runs, on three runs it was 40 μ-secs, on one 25 μ-secs, and on the remaining five runs, 4.5 μ-seconds. The "average" ratio of observed/casual rates was 35/1; the eleven events are therefore assumed to be genuine delayed events from which

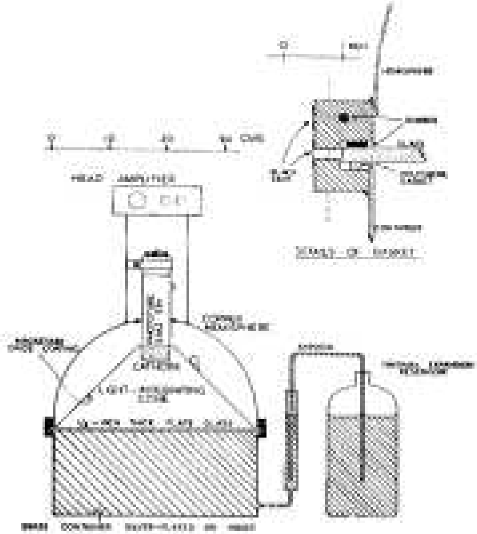

FIGURE 1

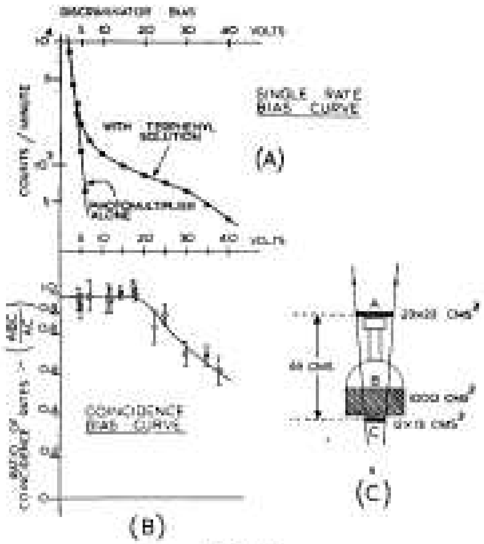

FIGURE 2

[after]

used to best advantage when the delayed and prompt events are observed from one and the same detector, thus eliminating all possible electronic delays.

(iii) Low background rates. It is possible with a scintillation detector, by using sufficient depth of the scintillation medium, to discriminate between the relatively low-energy events due to local radioactivity and the higher expenditures of energy in cosmic-ray tracks; this results in a much improved ratio for the useful counting rate relative to background, over that possible with a counter tray.

The detector in its present form.

In figure 1 is shown a diagram of the essential features of the detector. The brass cylindrical container, of area 1000 cm^2 and depth 15 cm. (corresponding to 23 MeV spent by a vertically incident relativistic particle), was optically polished and silver-plated on the inside. The container was filled with a solution of para-terphenyl in pure benzene (of "Analar" quality), 2 gms/litre, and sealed by a glass plate $\frac{1}{4}$" thick, using a polythene gasket $\frac{1}{4}$" in diameter. Details of the gasket are shown inset in figure 1. The light, collected in a detachable aluminium cone coated with fresh magnesium oxide powder, falls on the photocathode

UNDERGROUND EXTENSIVE SHOWERS

J.W. Sturgess

Birkbeck College, University of London

The existence of extensive showers of penetrating particles underground is reported. They have been recorded over a separation of 120 metres at a depth of 60 metres water equivalent. Threefold events may be explained as due to the hard component of extensive showers of primary energy greater than 10^{14} eV. Twofold events are much more frequent than expected from air showers and may be explained by the local production of penetrating pairs of particles.

1. Introduction.

For the past three years, observations have been made of coincidences between widely separated trays of counters at a depth of 60 m. water equivalent at Holborn Station. Except for their underground location, these measurements are similar to those made on extensive air showers by Auger and Maze (1938), Cocconi, Loverdo and Tongiorgi (1946, a, b) and McCusker (1950).

The apparatus consists of three banks of counters, each of area 3000 cm.^{2} and each bank consists of three closely packed trays of counters, fifteen in each tray. The three trays are arranged in threefold coincidence.

level (predominantly μ-mesons) were selected by the two Geiger trays A and C between which the scintillation detector B was situated. The ratio of the coincidences (ABC/AC) was measured at various discriminator bias voltages. The resulting bias curve, shown in figure 2(b), reveals that the detector has an efficiency in the central region of the container of (98 ± 2) % over a range of 0 to 18 volts on the particular bias scale. This corresponds to a range of from $> 10^4$/min. to 900/min. for the counting rate of the scintillator alone, as shown in figure 1(a).

Thus, if the detector is run at a bias of say 10 volts, half-way along the plateau, the single rate is only 1200/min. With this counting rate there is then a probability that 1 in 5×10^4 time-base sweeps (of duration 1 μ-sec.) will contain a casual pulse which might be mistaken for a genuine delayed event. (This pulse would be in addition to one always present near the start of each sweep).

Under these operating conditions the counting rate due to radioactive background was ~ 0.17 of that due to cosmic rays while, for a Geiger tray of the same area, the corresponding figure was 2.3 a factor of 12 in favour of the scintillator.

The most reasonable assumption is that the underground showers are the remains of extensive penetrating showers after the soft and nucleon components have been absorbed in the earth above. We shall assume that the particles giving rise to the events are mu mesons.

On this basis, it is of interest to compare the observations with those of McCusker and Millar (1951) using heavily shielded trays at sea level and with Greisen et al (1951) using threefold coincidences at 1600 m. water equivalent.

The actual values obtained are given in Column 4 of Table 2.

Table 2

Observers	Depth m.w.e.	Area of Trays cm^2	Rate day^{-1}	Corrected Rate day^{-1}	k
McCusker & Millar.	10(S.L.)	1750	2.8	6.2	0.75
Birkbeck	60	3000	0.44	0.44	0.17
Greisen et al	1600	7500	0.013	0.0033	0.0066

For comparison the rates have all been reduced to a standard area of 3000 cm^2 assuming the dependence on area to be proportioned to A^{γ} with $\gamma = 1.5$ (Cocconi, it is deduced that the upper limit to the fraction of triple coincidences which have associated delayed particles within the scintillation detector is 0.2 %, for delays greater than 0.5 μ-secs. and for showers of average density 20/metre^2. This result is not inconsistent with the findings of McCusker (Nature <u>166</u>, 400 (1950)) and of Mezzetti et al (Phys.Rev. <u>81</u>, 629, 1951)).

decrease much less rapidly than the total radiation. This would suggest that the meson spectrum for primaries of energy greater than 10^{14} eV is flatter than the mean spectrum. The energy of the primary may be calculated roughly because an event covering 100 m. will be part of a shower of area at least $10^4 m^2$. To be recorded in a tray of area 3000 cm^2 there must be at least 3 particles per square metre. The minimum energy required by a particle to reach a depth of 60 m. water equivalent is about 10^{10} eV, so that the total energy of the penetrating particles of the shower must be greater than 3.10^{14} eV.

An alternative explanation is that the particles are locally produced. However, at 60 m. water equivalent the apparatus is under 30 m. of earth. At 120 m. separation, secondaries of energy greater than 10^{10} eV would have to be emitted at angles greater than 60°. This has not been observed. Greisen in his work at 1600 m. water equivalent selected only those particles which were parallel which would prohibit local production

(ii) TWOFOLDS.

The results obtained in Holborn are given in Table 3.

particles the ratio of threefold to twofold events is

$$\frac{C_2}{C_3} = \frac{2^{\gamma} - 2}{3 \times 2^{\gamma} - 3^{\gamma} - 3}$$

following Ise and Fretter (1949). This function is plotted for various values of γ in Figure 2. For $\gamma = 1.5$, C_2/C_3 is approximately 3. But in the underground events $C_2/C_3 = 9.4/0.44 \simeq 20$. This would correspond to a value of $\gamma = 1.9$. However, the curve is tending rapidly to infinity here at a singularity. This region corresponds to low values of the density where the Cocconi spectrum has not been checked experimentally and the validity of the ratio should be treated with reserve.

The local production of penetrating pairs of particles has been established by several workers (Braddick and Hensby (1939); Braddick, Nash and Wolfendale (1951); George and Trent (1951)). It is likely that the high rate of twofolds can be explained by this mechanism. This will be more fully discussed by Dr. George in his paper on the interactions of mu-mesons.

Confirmation of the abnormally high ratio of twofolds to threefolds may be obtained from the results of Greisen et al., (1951) where at 6 m. separation

$$\frac{C_2}{C_3} = \frac{0.22}{0.013} \sim 17$$

2. Results

Threefold and twofold coincidences were recorded between the counter tanks at various separations. These may be dealt with separately.

(i) THREEFOLDS

The results obtained are given in Table 1.

Separation between extreme trays (m)	Coincidences	Time (days)	Rate/day
6	79	159	0.57 ± 0.06
20	101	165.5	0.61 ± 0.06
120	35	344	0.102±0.017

At six metres separation the trays were covered by 10 cm. of lead and the recorded rate was 0.44 ± 0.08 day^{-1} which is only slightly less than the rate without lead and is within the statistical error. This shows that the great majority of the particles recorded are penetrating.

This effect may be contrasted with sea level results when covering the trays with 10 cm. of lead caused a reduction of counting rate of 300:1 (Cocconi, Loverdo and Tongiorgi (1946 b)).

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Loverdo and Tongiorgi (1948 a)). The corrected rates are in column 5 of Table 2.

We may then obtain an indication of the absorption of the penetrating particles in extensive air showers in the earth.

A reduction of the shower density by a factor k, relative to sea-level, at a depth d underground will cause a corresponding reduction in the counting rate given by

$$\frac{R_0}{\bar{R}_d} = \left(\frac{1}{k}\right)^{\gamma} \qquad (1)$$

where R_0 = counting rate at sea-level

$\bar{R}_d$ = counting rate at depth d underground.

The last column of Table 2 gives the values of k derived in this manner. From equation (1) it may be seen that the value of k at sea-level should be, strictly, unity. McCusker (1950) has shown, however, that approximately 25% of the penetrating particles at sea-level are nucleons. The value of k at sea-level has been adjusted in view of this and the values of k refer to the mu-mesons only.

These values of k are plotted in Figure 1, which also shows the depth-intensity curve of the hard component obtained by drawing a line through the experimental points.

The rate of extensive shower particles is seen to

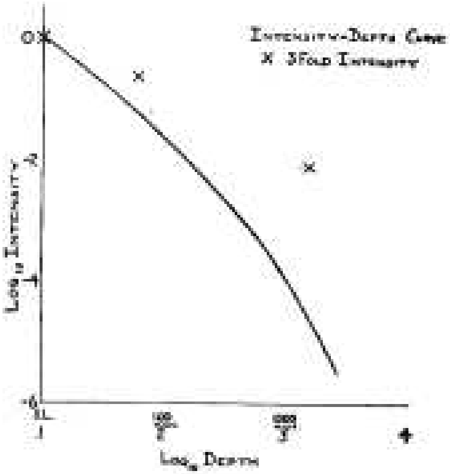

Figure 1

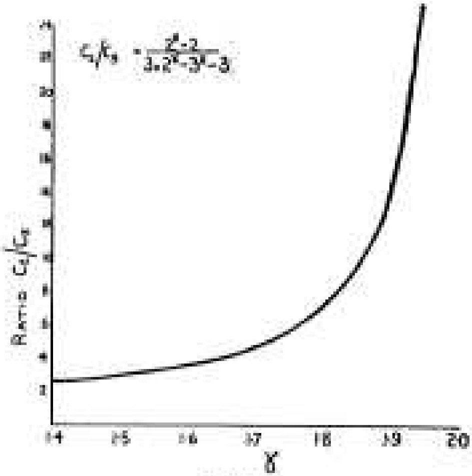

Figure 2

Sturgeon]

Table 3

Separation (m.)	Unscreened Rate day^{-1}	Screened 10 cm. Pb Rate day^{-1}
2.8	33.5 ± 1.9	23.2 ± 1.5
3.7	14.7 ± 1.1	10.8 ± 1.1
5.8	12.7 ± 1.1	9.4 ± 0.9
11	8.5 ± 0.9	-
15	9.6 ± 0.6	8.6 ± 1.3

From the table it will be seen that the screening effect of the lead is greater at small separations than at large. This indicates that some knock-on events are included in the unscreened rates at small separations.

The interesting feature of Table 3 is the high rate of twofold events. It is greater for example than the corresponding threefold rate at 6 m. separation at sea level of McCusker & Millar (1951).

There are two possible explanations of this : -

(a) γ is different from 1.5 for the penetrating components of extensive showers.

(b) Local production of pairs of penetrating particles.

For a Cocconi type density spectrum (1946 a)

$$N(>x) \propto \frac{1}{x}^{\gamma}$$

where $N(>x)$ is the number of showers with more than x

THE DECAY SPECTRUM OF μ-MESONS

C. O'Ceallaigh
H.H. Wills Physical Laboratory, Bristol and University College, Cork.

A. Theoretical Investigations.

In addition to its purely phenomenological interest, the form of the spectrum of the electrons from μ-decay has received much study in order to decide whether μ-decay and β-radioactivity may both be explained in terms of the same direct interaction between the particles concerned.

It now appears most probable that the decay of the μ-meson takes place in accordance with the scheme

$$\mu \longrightarrow e + 2\nu \qquad (1)$$

where ν represents a 'neutrino', i.e. a fermion of small or vanishing rest-mass.

The adoption of this scheme rests upon the following observations. Since the energy of the decay electron is not unique, it follows that it must be accompanied by at least two neutral particles. The fact that the average energy is about 1/3 $m_\mu c^2$, and that the end-point of the

3. Conclusion.

(i) Events have been recorded underground which can be interpreted as due to the hard component of extensive air showers with a primary energy of at least 10^{14} eV. The particles causing the events are probably mu mesons.

(ii) The rate of the extensive showers decreases less rapidly than the intensity of the hard component indicating that the spectrum above 10^{14} eV is flatter than the mean spectrum.

(iii) Many more twofold events are recorded than are expected from extensive shower events. These may be explained in terms of locally produced pairs of penetrating particles.

4. Acknowledgments.

This work was carried out under the supervision of Dr. E.P. George whom I wish to thank for many valuable discussions. I would also like to thank the London Transport Executive for making the excellent facilities at Holborn Underground Station available.

form of the interactions, whether scalar (S), vector (V), tensor (T), pseudovector (A), or pseudoscalar (P), has negligible influence on the predicted value of the decay-constant or the allowed spectrum shape. It is quite different in the case of μ-decay. Because of the relativistic velocities of the neutral particles, the form of the spectrum is profoundly influenced by the type of interaction assumed.

Calculations of the shape of the electron momentum spectrum for certain particular couplings, have been made by Tiomno, Wheeler and Rau[3]. The coupling constants for each assumed interaction were determined from the value of the half-life of the μ-meson, and were compared with the corresponding values for β-decay assuming a value of 30 mins. for the half life of the neutron. The coupling constants required for the various interactions were found to be of the same order for both phenomena. These conclusions are not altered by the shorter value (18 mins) now accepted for the neutron half life, the effect of this being to promote even better agreement. This fact is favourable to the assumption that the same theory may well apply both to β and to μ-decay.

Tiomno, Wheeler and Rau[3] consider three simple forms of the theory which they symbolise as follows,

Michel[4], and the results have been given in the following simple form, very convenient for comparison with experiment. Suppressing negligible terms, he finds

$$T_{\mu}\, P(E) = \frac{4\,E^2}{4} \left[3(W-E) - 2/3\ \rho\ (4E-3W)\right] \qquad (2)$$

where T_{μ} = Mean life of the μ-meson = $2.15 \pm 0.07 \times 10^{-6}$ sec, E = electron total energy, and W = energy at the end-point = 52.5 MeV ($m = 210\, m_e$). ρ is a parameter such that $0 \leqslant \rho < 1$ if the two emitted neutrinos are distinguishable (ν and $\bar{\nu}$), or $0 \leqslant \rho \leqslant \frac{3}{4}$, if the two neutrinos are identical (Majorana particles). The value of the parameter ρ is given by quadratic expressions in the coupling constants g_j; where g_1 corresponds to scalar (S), g_2 vector (V) ..... g_5 pseudoscalar (P), following the order for these interactions already quoted.

If only one $g_j \neq 0$, the normalizing condition $\int_1^W P(E)\, dE = 1/T_{\mu}$, yields the value of the selected g_j which is found to be of order 10^{-49} erg cm^{-3}, that of the Fermi constant for β-decay.

Fig. 1 shows the form of these theoretical spectra for selected values of ρ_2 plotted as a function of $E/W = u$. That for $\rho = 1$ corresponds to pure tensor interaction when only $g_3 \neq 0$, and for $\rho = 0$,

such captured μ^--mesons will contain electrons of energy in excess of the maximum energy for the decay of a free meson at rest. There will be a corresponding deficiency of electrons in the energy region less than the end-point. Porter and Primakoff[6] (1951) have given the results of calculations for the cases of Tiomno et al, A.C.E. - tensor (ρ = 0), and C.R. - vector ($\rho = \frac{3}{4}$). The neutrinos are treated as being Majorana particles. The corrections to the μ^+ spectra are not of very simple functional form. They depend on the magnitude of a parameter Z/137 and increase with increasing Z. A comparison between the μ^+ and μ^- spectra for $\rho = 3/4$ and Z/137 = 0.1 is given in Fig. 2, and applies with sufficient approximation to the case of G5 emulsion.

Thus, accurate experimental study of the form of the μ-decay spectrum, together with a precise value of the neutron half-life, may be expected to yield evidence on the question of whether the same direct interaction may explain both β -radioactivity and μ-decay. The problem of determining the nature of the interaction may be solved by an experimental measurement of the value of Michel's parameter ρ , but due allowance must be made for the operation of the Doppler smearing effect for μ^-mesons in circumstances where it spectrum lies at about $\frac{1}{2}\, m_\mu c^2$, points to the emission of just two neutral particles, each of negligible rest mass. It is reasonable to assume that they are identical. Counter coincidence experiments by Hincks and Pontecorvo[1], and by Piccioni[2], appear to exclude the possibility that they are photons. It is customary, therefore, to identify ν with the neutrino of β-decay. The spin of μ must be half-integral, whether that of ν is integral or half-integral. No experimental evidence is known which is inconsistent with the above assumptions. If, therefore, we accept the decay-scheme (1), we shall have to do with four fermions, and the interactions between them form the basis of the existing theory of the decay process.

There are good reasons for rejecting decay-schemes of the type $(\mu, \nu) - \pi - (e, \nu)$ which describe (1) as taking place through the intervention of a virtual π-meson. It is found that such schemes predict lifetimes many times greater than the observed value. For this reason, it has been usual to assume direct coupling between the four fermions μ, e, ν and ν.

It has been the aim of theoretical physicists to predict the shape of the decay spectrum by assuming that there exists a direct analogy between radioactive β-decay and μ-decay. Hence, the interactions involved will be those of the Fermi theory. In the case of β-decay, the

b) cosmic-ray mesons of mixed sign. (Davies Lock and Muirhead[13], Levi Setti and Thomasini[14], O'Ceallaigh[15]).

Cloud-Chamber Investigations.

The experimental arrangement of Lagarrigue and Peyrou[10], Fig. 3 will be taken as being typical, and differs from the others only in minor details. A rectangular chamber of useful volume 50 x 30 x 7.5 cms. was situated in a magnetic field of 3750 oersted. It contained two graphite absorbers B_1 and B_2 of superficial mass 2.65 gm/cm^2, and a tray (A, C_3) of four cylindrical counters enclosed in plexiglass. Above the chamber is a lead screen 20 cms. in thickness designed to absorb the greater part of the soft component, and between this and the chamber, are two banks of counters C_1 and C_2. Below the chamber is another bank of counters C_4 covered by a screen of 16 mm. of lead. The purpose of the screen is to minimise the possibility of a counter in C_4 being discharged by an electron from a decaying meson. The counters are arranged so that the chamber was expanded by the coincidence-anticoincidence scheme C_1, C_2, C_3-C_4. The efficiency of the arrangement was such that 0.6% of expansions, or about 1 per day, show μ-mesons stopp-

(a) Antisymmetrical theory with charge exchange (A.C.E.)

$$\begin{pmatrix} \mu^+ \\ \bar{e} \end{pmatrix} \rightarrow \begin{pmatrix} \nu \\ \nu \end{pmatrix}$$

where $\mu \rightarrow$ neutrino (ν), giving its charge to an electron in a negative energy state yielding a second ordinary neutrino, and leaving a positron $\bar{e}$.

(b) Simple theory with charge exchange (S.C.E.)

$$\begin{pmatrix} \mu^- \\ \bar{\nu} \end{pmatrix} \rightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}$$

where $\mu \rightarrow$ neutrino (ν), and gives its charge to a neutrino in a negative energy state, thus transforming it into an electron.

(c) Charge retention theory (C.R.)

$$\begin{pmatrix} \mu^- \\ \bar{\nu} \end{pmatrix} \rightarrow \begin{pmatrix} e \\ \nu \end{pmatrix}$$

where $\mu \rightarrow e$ and, simultaneously, a neutrino is raised from a state of negative to one of positive energy.

The results of these calculations for each of the three schemes are exhibited graphically for each of the types of coupling S, V, T, A and P listed above.

Calculations based on the general coupling scheme for four half-spin particles have been carried out by which emerge with little loss of energy. The action of the magnetic field is such that the positrons will be focussed on a stable circular orbit. Along portion of this orbit are disposed four anthracene crystals the signals from which are arranged to feed into a quadruple coincidence selector. To avoid background, the counting was carried out during an interval 3 - 10 μ sec. after the proton beam pulse. Calibration of the magnetic field then gave the form of the spectrum by plotting coincidence-rate against electron momentum.

(3) Multiple Scattering Technique in Photographic Emulsion.

As has been seen, the efficiency of collection of results by counter-controlled cloud-chambers is not high, so, to date, those of the greatest statistical weight have been obtained by determining the energy of the electrons by the technique of multiple scattering. This has already been described, and the distribution of the estimate of energy derived from finite samples of track has been discussed briefly. Owing to the difficulty of obtaining long tracks, the number of angles measured per track is usually not great, so that the estimate of energy of any individual track is subject to substantial statistical error. In order, therefore, to interpret correctly the experimto the case when either only $g_1 \neq 0$ or $g_5 \neq 0$, scalar or pseudoscalar interaction. To the degree of approximation chosen, all the curves pass through the same common point P.

Spectrum for μ Spin = 3/2.

The theoretical spectra so far considered, are based on the assumption that the spin of the μ-meson = $\frac{1}{2}$. The process $\mu \rightarrow e + 2\nu$ merely requires that the spin of μ be half integral. The effect of assuming spin = 3/2 for the μ-meson has recently been considered by Caianiello[5], and the theoretical spectra have been computed for the cases and couplings treated by Tionno et al[3]. Unlike the case of the spin $\frac{1}{2}$ μ-meson, it is found that none of the theoretical spectra go to zero at the end-point of the spectrum.

Doppler Smearing of μ-Decay Spectrum.

As is known, practically all μ^--mesons are captured into the lowest Bohr orbits about the nuclei of the atoms of the absorber. Once in a K-orbit, its subsequent fate will depend on two competing processes, capture by the nucleus with probability depending on Z^4 [6], or decay to an electron. For elements of $Z \leqslant 10$ decay is the more probable process[6,7]. Owing however, to the orbital motion of the meson, the decay energy-spectrum of in microns. Of these, some 90% say n, may be taken to be statistically independent. We may then show, if the ϕ_j s are distributed normally, an approximation sufficient for our purpose, that the experimental energy values for the individual tracks will have the distribution[16]

$$P(q)\,dq = \frac{2(\tfrac{1}{2}n)^{\frac{1}{2}(n-1-\alpha)}}{\Gamma\tfrac{1}{2}(n-1-\alpha)}\, q^{-(n-2+\alpha)} \exp(-\tfrac{1}{2}\, n\, q^{-2})\,dq \qquad (3)$$

where $q = E/\epsilon$. E = apparent, ϵ = true energy.

The value of the constant α will depend on the statistic employed to estimate the energy of the track. We have already considered the statistics, s = standard deviation of ϕ_j of sample, s_0 = mean square deviation from assumed mean = 0, and m_0 = mean deviation from assumed mean = 0. The following are the corresponding values of α .

Table I

Statistic	α
s	2
s_0	1
m_0	0.58

may be expected to produce measurable difference between the spectra for μ^+ and μ^- mesons. In principle, if $\frac{3}{4} \leqslant \rho \leqslant 1$, the neutrino must be a Dirac particle. If $0 \leqslant \rho < \frac{3}{4}$, this question cannot be decided from the form of the spectrum. Unfortunately, experimental evidence is such that this is, indeed, the case.

B. Experimental Investigations.

I. Methods.

In the principal investigations of the form of the μ-decay spectrum, three distinct techniques have been used. They are

(1) Measurement by magnetic curvature of the momentum of the tracks of electrons produced in the decay in absorbers in a cloud chamber of cosmic-ray μ^+ and μ^- mesons. (Leighton, Anderson and Seriff[9], Lagarrigue and Peyrou[10])

(2) Momentum measurements of electrons arising from the decay of artificially produced μ^+-mesons in a specially designed spectrometer. (Sagane, Hubbard and Gardner[11]).

(3) Energy measurements by the technique of multiple scattering, of electrons from the decay in Ilford G.5 emulsions, of

a) artificially produced μ^+ mesons (Bramson and Havens[12]).

sample of track. Since we have $N(\epsilon)d\epsilon$ electrons in the spectrum having energies $\epsilon \pm \frac{1}{2}d\epsilon$, the contribution of these, and only these, to the interval $E \pm \frac{1}{2}dE$ of the measured spectrum will be

$$d\left[\nu(E)\right] dE = N(\epsilon)d\epsilon\left[\frac{2\Psi(N,\alpha)}{\epsilon}(\epsilon/E)^{N-2+\alpha}\right.$$

$$\left.\exp\left[-\tfrac{1}{2}N(\epsilon/E)^{2}\right] dE\right] \qquad (5)$$

The contribution of the complete spectrum of all ϵ comprised $0 \leqslant \epsilon \leqslant \epsilon$ to the interval $E \pm \frac{1}{2}dE$ in the measured spectrum will be

$$\nu(E)dE = dE\int_0^{\epsilon_m} N(\epsilon)\frac{2\Psi(N,\alpha)}{\epsilon}(\epsilon/E)^{N-2+\alpha}$$

$$\exp\left[-\tfrac{1}{2}N(\epsilon/E)^{2}\right] d\epsilon \qquad (6)$$

We may transform to an expression containing the assumed spectrum in the standardized form

$$\nu(s)\,ds = 2\Psi(N,\alpha)\,s^{-N}\int_0^1 N(u)\,u^{N-3+2}\exp(-\tfrac{1}{2}N\,u^2 s^{-2})$$

$$du \qquad (7)$$

where $s = E/\epsilon_m$ and $u = \epsilon/\epsilon_m$.

-ing in the screens C_1 or C_2 or the counter tray A, the decay electrons from which were suitable for measurement.

Since the paths of the decay electrons were of very diverse geometry it was necessary to avoid any possibility of energy bias in the selection of tracks for inclusion in the statistics. This was done by applying the following selection criterion. An electron is accepted if, and only if, all electrons following the same path, of whatever energy E, $0 \leqslant E \leqslant 54$ MeV, could be measured with a precision in excess of a certain minimum value fixed in advance. The mean precision of the measurements was $\pm$ 6.5% .

(2) Spiral-Orbit Spectrometer[11]

In this instrument, the proton beam from the 184" Berkeley cyclotron was electrostatically deflected to pass through a 1¼" collimating hole drilled on the axis of one of the pole-pieces of a magnet. Between the pole-pieces, and coaxial with the field, were placed targets consisting of cylindrical rods of suitable material. The π^+-mesons produced in the proton bombardment decay to μ^+ mesons. Owing to a combination of circumstances, most of the μ^+-mesons will themselves decay within the target material to positrons tained by means of the various methods. Some evidence has been put forward which would indicate a difference between the spectra found for cosmic-ray mesons of mixed signs and for artificially produced μ^+-mesons.

Experimental Spectrum of μ^+-Mesons.

Preliminary results found by means of the spiral-orbit spectrometer by Sagane et al.[11] are as follows. The mode of the electron momentum-spectrum is found to lie at $p = 70 \pm 3\ m_ec$, and the intensity at the end-point approaches zero and appears to be less than 1/10 of the intensity at the mode. The results are interpreted as being best in agreement with that corresponding to Michel's parameter $\rho = 0$, or its equivalent, A.C.E - Tensor; C.R.-Scalar of Tiomno et al. The possibility is not excluded that the end-point intensity is as great as 0.1 times the model intensity, a result which could be explained by an appropriate linear combination of coupling constants e.g. in the S. C.L. theory of Tiomno, Wheeler and Rau. The calibration of the apparatus is at present under examination in order to establish the behaviour of the spectrum in the neighbourhood of the end-point.

Since the total number of tracks available from the work of Bramson and Havens is only 117, it is difficult

-ental results, a clear distinction must be drawn between the true spectrum and its 'distorted image', the experimental spectrum as measured by any technique of low precision.

For example, since we know that there is a 'cut-off' in the energy spectrum at about 54 MeV, (at least for the μ^+-spectrum), the first effect of statistical spread will be the appearance of electrons with apparent energy $E > 54$ MeV accompanied by a corresponding distortion in the region $0 \leqslant \varepsilon \leqslant 54$ MeV. These considerations will be clear from Fig. 4 which is sketched to illustrate the effect for a hypothetical spectrum. It is to be understood that the area under each curve must be the same. In this extreme case, it is clear that a spurious mode appears in the experimental spectrum, although the true spectrum is monotonic increasing. We seek, therefore, the transformation True Spectrum $\rightarrow$ Measured Spectrum.

Suppose that an experiment is carried out in which the energy of a very large number of electrons, all of the same energy and track-length, .. microns is estimated by measuring the multiple scattering of each, using cell-size c microns. As already shown, the number of values of ϕ_j available for this purpose is $\nu = L/c - 1$, where L is the track length

Table II

Experiment	No. of Tracks	Estimate of ρ
(1) L.A.S.	75	0.75 ± 0.20
(2) L.P.	150	0.23 ± 0.15
(3) (1) + (2)	225	0.19 ± 0.12
(4) L.P. e^+	121	0.05 ± 0.18
(5) L.P. e^-	104	0.35 ± 0.19

It is difficult to draw more than tentative conclusions from these figures. As pointed out by the authors the experimental difference between the μ^+ and μ^- spectra could arise from statistical fluctuation with probability 18 % and cannot, therefore, be accepted as significant. The apparent difference in the behaviour of the e^+ and e^- spectra near the end-point, is not that to be expected if the Doppler smearing effect of Porter and Primakoff were operative. This effect will be of less importance in cloud-chamber work where the parameter Z/137 is $\sim$ 0.5 as against $\sim$ 0.1 for the photographic emulsion. Owing however, to the difficulty of collecting statistics, it is scarcely feasible to test

The value of α for the statistic most commonly used (m_o) is approximate, as the expression for the sampling distribution of energy estimated by this statistic is of very inconvenient form

The above theory has been tested experimentally[16] using samples of track of equal length of monoergic electrons, and the agreement of experiment with theory has been found to be very satisfactory.

Consider, next, any theoretical spectrum, and suppose that there are $N(\epsilon)d\epsilon$ electrons comprised within the energy-interval $\epsilon \pm \frac{1}{2} d\epsilon$. In determining the experimental spectrum, all tracks are chosen to have the same length and are measured in the same cell-size regardless of their energy, so that there are n independent cells in each. Rewriting (3) we have

$$P(E)\, dE = \frac{2\,\Psi(n,\alpha)}{\epsilon} \;(\epsilon/E)^{n-2+\alpha} \exp\left[-\tfrac{1}{2}n(\epsilon/E)^2\right] dE \tag{4}$$

where $\Psi(n,\alpha)$ is written for $\dfrac{(\frac{1}{2}n)^{\frac{1}{2}(n-1-\alpha)}}{\Gamma\frac{1}{2}(n-1-\alpha)}$, and $P(E)dE$ is the probability that an electron of which the energy is ϵ, will appear on measurement to have energy comprised within $E \pm \frac{1}{2}dE$, there being n statistically independent values of ϕ_j in the energy at the end-point, and a knowledge of the maximum energy release based on the known mass of the μ-meson.

For these reasons, much experimental and theoretical work has been carried out in order to determine the value of the scattering constant, and its variation with particle velocity. This constant, K , is defined as follows[18]

$$\bar{\alpha}_s = \frac{K_{s,\beta}\, s^{\frac{1}{2}}}{p\beta c} = \frac{K_{s,\beta}\, s^{\frac{1}{2}}}{M_0c^2\, B\beta^2} = \frac{K_{s,\beta}\, s^{\frac{1}{2}}}{T\left(\frac{2+\sigma}{1+\sigma}\right)} \qquad (9)$$

where p = momentum (MeV/c); M_0c^2, T, rest and kinetic energy (MeV); $B = (1-\beta^2)^{-\frac{1}{2}}$; $\sigma = T/M_0c^2$, of the particle of velocity $= \beta c$. s = cell-size in units of $100\,\mu$.

Theory shows that the value of the scattering constant may be written

$$K = 8.203\, L_{s,\beta}\ \frac{\text{degrees x MeV}}{(100)^{\frac{1}{2}}} \qquad \text{(G5 emulsion)}^{18}$$

For the electrons of the μ-decay spectrum, the value of $\beta^2 \sim 1$, we need only consider the variation of $L_{s,\beta^2\sim 1}$ with cell-size. For $\beta^2 \sim 1$, this is very well represented by the empirical expression[18]

$$L_{s,\beta^2\sim 1} = 1.45 + 1.19\,(\log t)^{\frac{1}{2}} \qquad (10)$$

t = cell-size in microns.

On substituting Michel's general expression for $N(u)$ we have,

$$N(u) = 4/3 \left[(9 - 6\rho)u^2 - (9 - 8\rho)u^3 \right]$$

following some reduction, we obtain the following general expression for the standardized measured spectrum.

$$N(s)\,ds = 4/3\,(2/n)^{3/2}\,s^2 \Bigg[(9-6\rho) \frac{\Gamma \frac{1}{2}(n-\kappa+4)}{\Gamma \frac{1}{2}(n-\kappa+1)}$$

$$I\left(\frac{\frac{1}{2}n\ s^{-2}}{(\frac{1}{2}n-\kappa+4)}\ \frac{1}{2},\ \frac{1}{2}(n-\kappa+2) - (2/n)^{\frac{1}{2}} \frac{(9-6\rho)s}{\Gamma \frac{1}{2}(n-\kappa+1)} \right)$$

$$I\left(\frac{\frac{1}{2}n\ s^{-2}}{\frac{1}{2}(n- \quad +5)},\ \frac{1}{2}(n- \quad +3) \right) \Bigg] \qquad (8)$$

The expressions $I(p,q)$ are incomplete Γ-functions in the standard form tabulated by Karl Pearson[17].

II. Experimental Results.

As we have seen, the aim of the experimental investigations is to estimate the value of Michel's parameter ρ, which is a function of the interactions between the four half-spin particles with which we have to deal. At the time of writing there appears to be no very satisfactory agreement between the results obplate method, was that of Davies, Lock and Muirhead[13] who used the Kodak N.T. 4 emulsions. This work established the fact that the decay energy of the electrons was not unique and showed that the average energy was $\sim 1/3\ m_{\mu} c^2$. Owing to the small number of tracks available (81), it was not possible to draw more than the tentative conclusion that ρ might exceed zero. The general appearance of their curve agrees, within the limits of statistical error, with those found subsequently by the same method.

The work of O'Ceallaigh[15] seems, at present to be that of greatest statistical weight. The scattering of the secondary particles from some 700 'ρ'-mesons has been measured. The results have been divided into three series, in each of which the tracks were of identical length. Details are given in Table IV

Table IV

No. of electrons in series.	Length of track (microns)	No. of cells (n) (75μ)	S.D. of estimate of energy.
I 688	1050	13	21.1%
II 380	1950	25	14.3%
III 218	2850	37	11.7%

In order to allow for possible uncertainty in to judge the agreement between their work and that of Sagane et al. Unfortunately, the distortion of the true spectrum by statistical uncertainty has not been taken into account by these workers. Further, the selection criterion employed was the acceptance of all tracks of length $\geqslant$ 1500 microns. Since the distribution of track lengths > 1500 microns is not available, it is not possible to carry out the necessary transformation except in a rough fashion. The published curve does not appear to be in disagreement with the hypothesis $\rho = 0$, but firmer conclusions must await improved statistics and technique.

Spectra of Mixed Cosmic-Ray μ-Mesons.

The statistical weight of the results of Leighton et al.[9] (75 tracks) is low, but they do not differ significantly from those based on the 150 measurements of Lagarrigue and Peyrou[10]. The latter authors have derived values of ρ both from the separate series, and from a combination of both sets. This procedure seems to be justified as the experimental arrangements do not appear to differ in any essential particular. These results are given in Table II.

meter ρ , and the goodness of fit with the experimental histogram was tested by means of the X^2 test.

The results may be summarised as follows. Taking the value $K_{100} = 24.3$, which as may be seen from Table III, is in satisfactory agreement with observation and theory, the experimental values could not be fitted by any choice of ρ, $0 \leqslant \rho \leqslant 1$. This result was found in each of the experimental series I, II and III, and those for II are shown in Fig. 6.

For $K = 24.3$, there is an excess of high-energy particles, even for $\rho = 1$. By choosing a lower value for K, it was found possible for certain values of to fit the experimental figures in a manner judged satisfactory by the X^2-test. The result of such choice is shown in Fig. 7 which also refers to series II. The broad band shows the region of acceptable fit ($\geqslant$ 5 % probability) for $\rho = 0$, and is bounded by the transformed curves for $K_{100} = 20.5$ and $K_{100} = 19.3$. The 5 % level is that conventionally adopted by statisticians as being 'significant', and means that in only 5 % of cases could a value of X^2 greater than the experimental value arise purely as a matter of chance. For purposes of comparison, the transformed spectrum is plotted for $\rho = 0$ and $K = 23.0$

The results of the fitting experiments for Series this point by using heavier gases to fill the chamber. It is, perhaps, a little surprising that so few electrons have been observed in any of the cloud-chamber experiments with apparent energies in excess of 54 MeV, since the average accuracy is about 8.5 % and statistical distortion effects might be expected to be operative of order of magnitude not different from those verified to occur in photographic emulsions.

Spectra of Mixed Cosmic-Ray Mesons in Photographic Emulsions.

The decay particles measured by Levi Setti and Thomasini[14], and O'Ceallaigh, 1951[15] are those arising from the decay of cosmic-ray 'ρ' -mesons. Calculation shows that if equal numbers of μ -mesons decay at rest in G-5 emulsions, then, of the observed electrons, some 60% will be e^+ , and 40% e^-. The value of the ratio μ^+/μ^- appropriate to the exposure may only be found indirectly from a study of the relative numbers of π and μ -produced events in the series of plates.

The interpretation of the experimental results obtained by the photographic plate technique is greatly dependent on a knowledge of the exact value of the 'scattering constant' appropriate to the experimental conditions. In the present case, it is clearly not possible to obtain a calibration from the observed highly unreasonable to suppose that its true value can be as low as 21. Thus, we may reject with confidence a μ-decay spectrum with $\rho = 0$. Since it is very improbable that K_{100} is as low as 22, it seems safe to conclude that no choice of the value of ρ consistent with reasonable assumption of the uncertainty of our knowledge of that of K_{100}, leads to acceptable agreement with experiment. In this connection it is relevant to note that the observers[21] who were responsible for the experimental determination of K_{100} for the 105 MeV positrons Table (II), also carried out the measurements of the μ-decay spectrum.

The general feature of the present experimental results is the presence of an excess of electrons with apparent energies greater than the spectral maximum. The category of meson styled 'ρ-meson', consists of all particles distinguishable from protons, which come to rest in the emulsion without producing a star. They include therefore π-mesons which produce 'zero prong' stars (some 30% of all σ-stars), all captured μ^--mesons, and such μ^+ and μ^--mesons as decay to electrons whether recognised or not. It is possible for a scanner to record as a ρ-meson any particle decaying to a secondary at minimum ionization. Among such might be κ-mesons or π-mesons, if direct decay to an electron following

In order to avoid undue statistical fluctuations in the value of $\bar{K}$ which might arise from the presence of one large plural or single scatter, it has been customary to exclude all values of $\varphi_j \geqslant 4|\overline{\varphi_j}|$, and if this is done, it is necessary to reduce the value of the scattering constant by 10 %. This reduced K we may denote by $K_{co.}$ and the chief theoretical and experimental values for the case $\beta^2 \sim 1$ are listed in Table III.

Table III

Authors	Method	$K_{co.}$ $\frac{0 \times MeV}{(100\mu)^{\frac{1}{2}}}$
Moliére[19]	Theory	23.0
Snyder[20]	Theory	24.3
Menon, O'Ceallaigh & Rochat[21]	105 MeV positrons 186 MeV positrons	24.6 22.5
Corson[22]	positrons 40-165 MeV	26.0
Bosley & Muirhead[23]	electrons 8.7-17.2 MeV.	24.3
Voyvodic & Pickup[24]	Electrons + and - from materialized 17.6 MeV γ-rays.	22.7

The pioneer investigation by the photographic could have been produced by the direct decay of unstable particles of mass > m_π.

There remains the possibility that the apparent excess of electrons with apparent energies greater than the spectral maximum is explicable by the Doppler smearing of the μ^- component, in the manner envisaged by Porter and Primakoff[8]. Indeed, the operation of the effect is in the right direction to bring theoretical spectrum into better agreement with experiment. Calculations are in progress which will enable this possibility to be tested, but they are tedious because of the functional form of Porter and Primakoff's expressions. When these have been completed it will be possible to judge if the effect is of sufficient magnitude to account for the discrepancy. Additional independent evidence may be expected to become available from the work of Bramson and Havens,[12].

As far as may be judged, the results of Levi Setti and Thomasini[14] are not in disagreement with those just described. In all, 368 tracks have been measured by these authors. They are divided into two series of which the main comprised all tracks with length exceeding 1 mm. The average length of these was 3.2 mm. but 2/3 were of length < 3 mm. A technique of preliminary measurement was used to choose an optimum cell-size for the final measurement. Thus it is a difficult the value of K_{100}, the experimental results were treated as follows. The number of observations (N) were plotted as a histogram for a selected range of $(\bar{\alpha})^{-1}$. The most suitable group width was found to be $\Delta(\bar{\alpha})^{-1} = 0.3$. The result for series I is shown in Fig. 5.

The value of $(\bar{\alpha})^{-1}_{m}$ corresponding to the end-point of the spectrum 53.8 MeV, ($m_{\mu} = 210\ m_e$) was computed as follows. The mean energy-loss along the course of the track was taken to be 1.4 MeV/mm. This figure assumes that radiation losses > 50 % would manifest themselves by a change in the trend of scattering along the track. The apparent energy of the track was assumed to be that corrected to the mid-point. Thus, for tracks of length L_{μ}, the apparent maximum energy was taken to be T_{eff} = (53.3 - 1.4 L/2000) MeV. We have, from (9), $(\bar{\alpha}_{100})^{-1}_{m} = K_{100}/T_{eff}\ \dfrac{(2+\sigma)}{1+\sigma}$. The transformed distribution curves (8), which are expressed in terms of $s = E/\epsilon_{m} = \dfrac{(\bar{\alpha}_{100})^{-1}_{m}}{(\bar{\alpha}_{100})^{-1}}$ were then fitted to the histograms of the three series (Table IV), taking into account the number of observations in each. This fitting was carried out for various assumed values of K_{100} and Michel's paramesons by the scattering method, but failing the same method of analysis it remains difficult to compare them more exactly. The writer acknowledges with gratitude much helpful correspondence with other workers in the field, in particular with Drs. Levi Setti and Thomasini, and Dr. Michel, and thanks them and the workers at the École Polytechnique for generous permission to quote from their unpublished work.

II are presented in a systematic fashion in Fig. 8. Here, the continuous curve is that of the experimental values of X^2 for $\rho = 0$, plotted as a function of the assumed value of K_{100}. The statistical levels of significance corresponding to the experimental number of degrees of freedom (9), are also shown, and the interpretation of the goodness of fit is indicated on the diagram. The points $\rho = \frac{1}{4}$ and $\rho = \frac{1}{2}$, mark the approximate positions of the minima of X^2 for these values of the parameter. The dotted curve is the envelope of these minima. Consideration of the figure leads to the following conclusions.

(1) For $K_{100} \geqslant 22$, the experimental results cannot be fitted by any ρ ,$0 \leqslant \rho \leqslant 1$.

(2) For $19.1 \leqslant K_{100} \leqslant 22.0$, it is possible to fit the results by choice of ρ , $0 \leqslant \rho \leqslant \frac{1}{2}$, but that the goodness of fit decreases with increasing .

These conclusions derived from Series II are supported in a satisfactory manner by those for Series I and III. This observation seems to be confirmatory of the correctness of the transformations (8).

The satisfactory agreement between theory and experiment as shown in Table III, makes it very difficult to accept a value of K_{100} as low as 22, and it appears

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the scheme $\pi^+ \rightarrow e + \nu$, is a serious competitor[25] to the process of $\pi - \mu$ decay. Indeed by assuming a sufficiently high probability for the direct-decay process, it is quite possible to fit the experimental results using the accepted value of K_{100}, but there are serious objections to this. In the first place, measurements by Friedman and Rainwater[26] on π^+-mesons ending in photographic plates in circumstances where they could be distinguished from background μ-decays, show that the probability of π-e decay / μ-e decay is $\sim$ 1/1400. Similar experiments tend to confirm this result. Having regard to the ratio $\pi - \mu$ decays/ ρ-e decays observed in our plates, the above probability is insufficient to explain the excess of high energy particles.

Furthermore, mass estimates by the technique of gap-length count vs. residual range were made on those ρ-mesons which produced minimum tracks of apparent $p\beta c > 50$ MeV. Even though there was a tendency to overestimate the mass of many of these, because of the appreciable dip of their paths through the emulsion, none of the selected ρ-mesons appeared to have mass in excess of that of the π-meson. It appears unlikely, therefore, that any appreciable fraction of the high-energy electrons

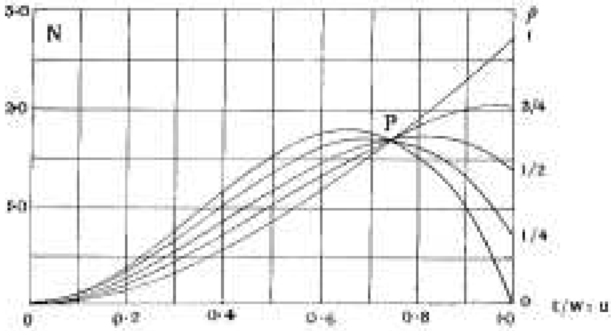

Figure 1

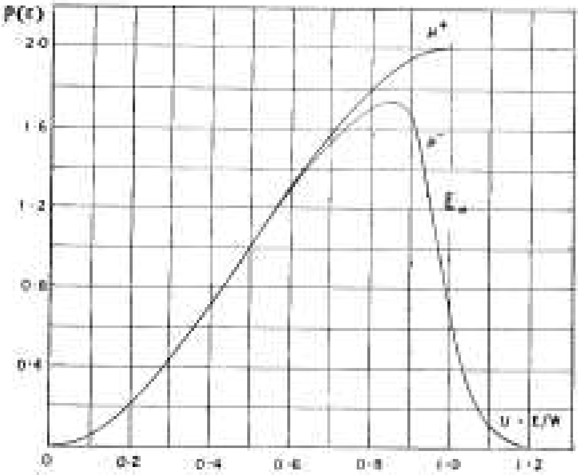

Figure 2

matter to compare the statistical spread of these results with those of O'Ceallaigh. The constant used is 26 which is higher that that assumed in the latter work. The results for the 278 tracks of length > 1 mm. are reproduced in Fig. 9. They are, perhaps, best compared with those shown in Fig. 5. Superficially, the agreement seems good, the positions of the modes agree and the spread beyond 53.8 MeV. the theoretical end-point is about the same in both. The spread in the high energy regions is largely determined by those tracks which are measured in the smallest number of cells. Since this information is not available to the writer, it does not seem profitable to draw further conclusions at the moment. Levi Setti and Tomasini also give a histogram for 368 tracks measured, for the most part on 10 cells. This is reproduced in Fig. 10. The general appearance accords well with that of series I of the present work (13 cells), (Fig. 5). For purposes of comparison the approximate position of the spectrum of Fig. 5 is shown in hatched line on Fig. 10. The position of the mode is about the same in both and occurs at a lower energy than in Figs. 6 and 9. In the light of the information at present available, there seems no reason to doubt that substantial agreement exists between the results for mixed cosmic-ray

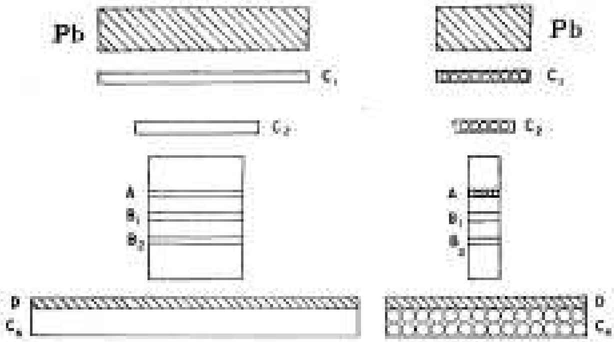

FIGURE 3

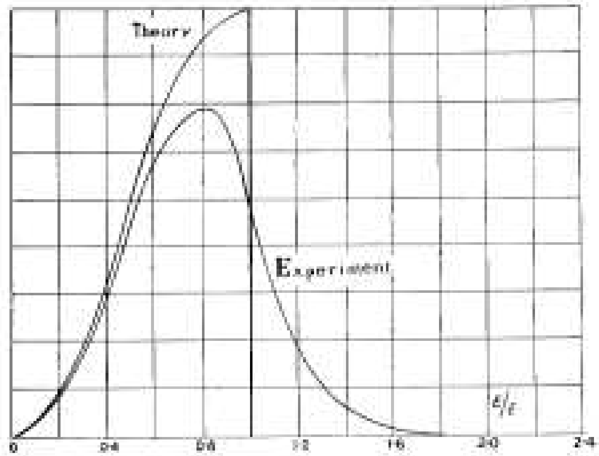

FIGURE 4

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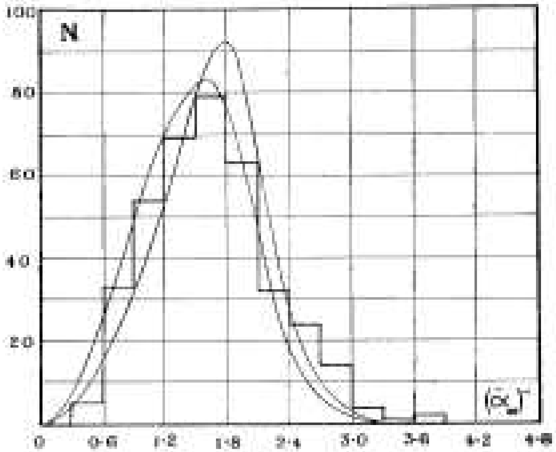

FIGURE 5

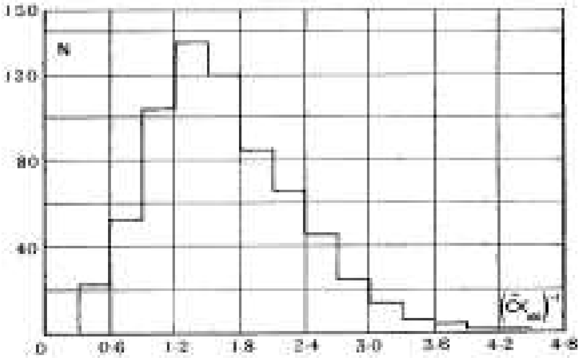

FIGURE 6

[Catchpole et al]

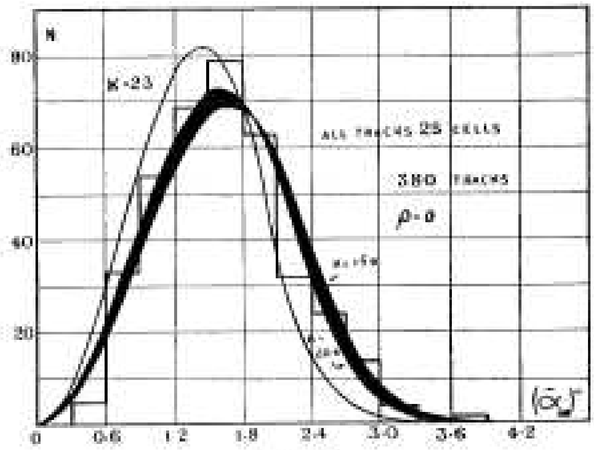

FIGURE 7

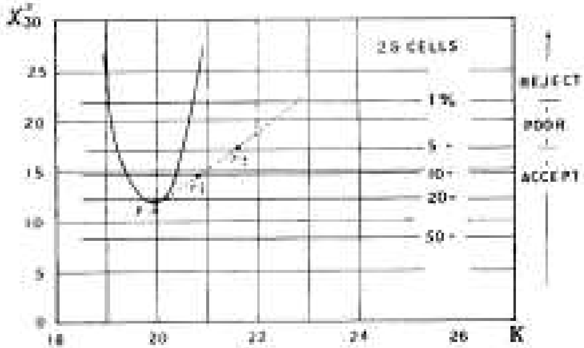

FIGURE 8

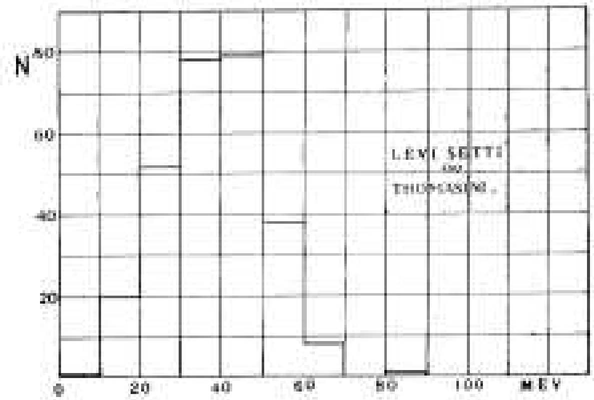

FIGURE 9

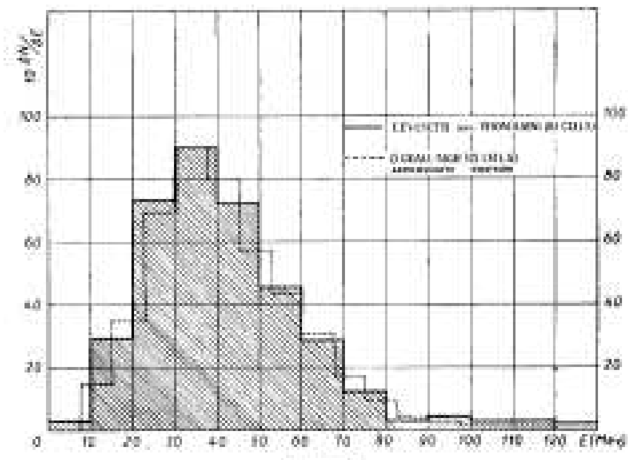

FIGURE 10

[illegible] a]

NUCLEON CASCADES IN LEAD

G.D. Rochester
The Physics Department, University of Manchester

§1. Introduction

There is now much evidence that the main build-up of cosmic rays in the atmosphere occurs through nucleon cascades in the way shown schematically in Fig. 1. Primary particles collide with air nuclei creating mesons and forming other fast nucleons which in turn move on to make other collisions. In these collisions further mesons are created and more nucleons formed. After a few collisions the energies of the nucleons are too low to create mesons, but the nucleons can still produce evaporation stars. The final products are μ-mesons, and slow protons and neutrons. The slow protons are absorbed by the loss of energy by ionization, and the slow neutrons in capture by nitrogen nuclei. The evidence for nucleon cascades is as follows:-

(1) The direct observation of secondary interactions in the cloud chamber and in the

as well as in air occurs through the mechanism of nucleon cascades.

§2. The Determination of the Interaction Mean-Free-Path of Charged N-Rays.

The interaction mean-free-path (sometimes termed the collision length), denoted by λ_I , is defined as the average distance traversed by an N-ray before it produces its first nuclear interaction. It is found that the variation of intensity of N-rays can be represented by an exponential function, i.e.

$$I = I_0 \exp(-x/\lambda_I) \qquad \ldots\ldots (1)$$

where I_0 is the intensity of the incident rays, I of the emergent rays and x the thickness of the absorber. The interaction mean-free-path corresponding to the geometrical cross-section of the target nucleus is given by

$$\lambda_I = 1/\sum n_j\ \sigma_j \qquad \ldots\ldots (2)$$

where n_j is the number of nuclei of type j per unit mass of the absorber and σ_j the cross-section of this nucleus. The values of λ_I for carbon and lead are about 60 g.cm^{-2} and 160 g.cm^{-2} respectively

Cocconi (1949) devised a counter arrangement for

for secondaries which can penetrate 200 g.cm^{-2} of absorber.

All the experiments seem to indicate that for high energy events in lead the interaction mean-free path is consistent with the value calculated from the geometric cross-section.

Experiments have also been carried out for carbon using the same apparatus as for lead, and they find 81 $\pm$ 5 g.cm^{-2}, a value which is slightly greater than that corresponding to the geometrical cross-section. This result, if correct, probably means that the carbon nucleus is more transparent than the lead nucleus. Transparency for the scattering of 90 MeV neutrons by light nuclei has already been found at Berkeley.

§3. The Interaction Mean-Free-Path of Neutral N-Rays.

The apparatus of Fig. 3 may also be used for this determination but now the counters of tray A are connected in anticoincidence with the shower set P. The values found by Walker et al. (1950) agree closely with those for charged N-Rays.

§4. Secondary Interactions in the Photographic Emulsion.

The value of λ_I can be found directly by observing secondary stars in the photographic emulsion. Six nucmesons.

§5. The Determination of the Attenuation Length for Condensed Matter.

The attenuation length, denoted by λ_A, is defined as the thickness of matter which reduces the intensity of N-rays by a factor 1/e. It is found experimentally that the attenuation of N-rays is exponential, i.e.

$$I = I_0 \exp(-x/\lambda_A) \qquad \ldots\ldots (3)$$

where I is the intensity of all N-rays, irrespective of their origin, at a depth x below the top of a layer of absorber, and I_0 is the intensity of the N-rays above the absorber. The value of λ_A is determined by measuring I as a function of x by a device which records N-rays e.g. a penetrating-shower set or a photographic emulsion. Values found for different materials by the photograph emulsion method are summarised in the Table 1.

photographic emulsion.

(2) The examination of the energies of the recoil fragments of nuclei in evaporation by Harding. These observations show that in the formation of a star a neutron is, in general, not stopped catastrophically but passes through the nucleus and goes on to form another star.

(3) Both the star rate and the slow neutron rate have maxima well below the top of the atmosphere.

(4) The magnitude of the latitude effect of stars.

(5) The comparison of the attenuation length of any nucleonic event with the interaction mean-free-path. In general, the attenuation length for all types of nucleonic events are greater, (usually about 1.5 - 2 times), than the interaction mean-free-path. This means that secondary interaction has taken place.

This qualitative picture has been given quantitative form in the work of Messel and his collaborators. All the main features are brought out in their theory; the numerical agreement depends upon a knowledge of the primary spectrum. A theory neglecting ionization loss has been given by Fujimoto and Hayakawa. It may be inferred that the main build-up of the nucleonic component of cosmic rays in dense absorbers

TABLE 2

Types of Stars in Air Plates at 2860 m.

$E_H \longrightarrow$

	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Total
0n	572	380	193	91	56	51	28	11	8	4	2	1	5	1	1		1							1405
0p	31	28	26	18	8	11	6	3	3	1	1	1		1	1	1		1			1			142
1n	37	19	21	15	8	7	6	3	2	1	2		1		1	1								124
1p	18	13	16	5	3	4	2	2			1							1						65
2n	1	2	5	3	4		3	1	2	1	2	1					1	1						27
2p	3	5	1		1	3	2			1		1		1										18
3n				1		1	1		1	1														5
3p	2			1	1	1			1												1			7
4n											1		1											2
4p	1				1																			2
5n			1	1																				2
5p				1																				1
>5n																					1(11n)			1
>5p		1 (6p)												1(7p)										2

To get the rate/c.c./day after correcting for the background multiply by ~ 5.0×10^{-3}.

the determination of λ_I which is shown in principle in Fig. 2. The counters are so arranged that the showers in P which are recorded are those for which only the counter in tray A is discharged. The actual arrangement used by Cocconi at altitudes of 3,260 m. and 260 m. is shown in Fig. 3. Counters B, C, and D, which record the penetrating showers, are each separated by 1 inch of lead. Coincidences of the type A + B + C + D + 1E are taken as a function of the thickness of absorber $\sum^1 + \sum$. Thus to produce a shower a ray has to set off only one counter in tray A and only one in tray E. After correcting for the knock-on shower background Cocconi finds $\lambda_1(Pb) = (160 \pm 15)gcm^{-2}$.

Similar experiments have been carried out by Walker (1950) and Sitte (1950). These authors have extended the Cocconi work by determining the mean-free-path for the charged primaries which produce showers with different numbers of relativistic secondary particles. Both workers find a decrease in the value of λ_I with increasing shower size. Sitte finds $\lambda_I(Pb) = (196 \pm 13)$ $g.cm^{-2}$ for secondaries which can penetrate 100 $g.cm^{-2}$ of absorber and $\lambda_I(Pb) = (162 \pm 10)$ $g.cm^{-2}$

TABLE 3

Types of Stars under 30 cms. of Lead at 2860 m.

$N_H \longrightarrow$

	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Total
0n	554	356	202	88	45	25	20	5	4	6	2	1		1	2			1						1312
0p	35	31	32	21	18	8	10	4	2	4	2	1		2			1							171
1n	24	22	14	10	4	8	7	3	1	1		2	1			1				1				99
1p	17	16	9	13	8	2	1	1	4	2		2				2								77
2n		2	2	2		1	2			1						1		1				1		13
2p	5	7	1	2	1	1	1		2	1		2		2	1						1			27
3n	1	1			2			1				1			1									7
3p	1	3	1		1				1		1			1		1					1	1		12
4n				1	1				1						1									4
4p													1										1	2
5n																1								1
5p				1						1														2
>5n														1(6n)						1(14n)				2
>5p														1(9p)		1(6p)								2

-lear interactions have been found by Camerini et al. (1950) in 128 cm. of π-meson track length in the emulsion, giving a value of $\lambda_I = 82 \pm 35$ g.cm^{-2} of emulsion. The value of λ_I for all thin tracks of length greater than 400 μ is 102 ± 27 g.cm^{-2}. Both of these values are close to the value expected from the geometrical cross-section - i.e. 90 g.cm^{-2}.

It is interesting to note the kind of stars produced in these secondary interactions. Of 40 stars produced by protons, eleven are in the energy range 600 - 850 MeV, and in only one case is an identified π-meson produced. From the consideration of the energy balance, there appears to be no evidence for "missing" energy. Of 40 stars produced by π-mesons, 22 are caused by mesons with energies of from 600 to 1,100 MeV. In six cases one π-meson also comes out of the star, and in five of these the energy of the emitted meson is about 4% of the incident π-meson. In the remaining case the emergent π-meson has half the energy of the incident meson. Considering the energy balance of all of the 40 stars, there is evidence of "missing energy. It is therefore possible that neutral π-mesons are produced as well as charged events with the value of λ_{I}.

(2) To find what fraction of stars under load can be ascribed to fast π-mesons whose mean-free-path for star production is less than the mean-free-path for decay into μ-mesons.

Ilford G5 plates, 300 μ in thickness, were exposed for 21 days at the Pic du Midi, altitude 2867 m., one group of plates being placed under 30 cm. of lead built in the form of a hemisphere, and the other group 1.5 m. away from the lead. Both groups of plates stood on the concrete floor of a hut with a thin roof. The transport of the plates resulted in a "background" equivalent to 3.3 % of all the stars observed. The choice of the thickness of the emulsion was the result of careful tests of the accuracy and speed of the observers. Scanning was carried out with 20 x Cooke objectives and 10 x eye-pieces.

Each star was examined in detail with a 45 x Oil-immersion objective and grain counts were made on the tracks. In order to be accepted for the analysis, the star had to have at least 3 prongs or 2 prongs and a nuclear fragment. The usual criterion was also adopted to exclude stars of radioactive origin. The stars were classified in the same way as by Brown et al. (1949) and the results are shown in Tables 2 & 3.

Table 1. The Attenuation Length (g.cm^{-2}) in Different Materials.

Authors	Material			
	Carbon	Ice	Aluminium	Lead
Barton, George & Jason (1951)	166 $\pm$ 7			
George & Jason (1949)				310 $\pm$ 20
Bernardini et al. (1949)			220	300 $\pm$ 20
Harding, Lattimore, Li & Perkins (1949)		200 $\pm$ 10		

These values were determined for the most part by using Ilford C2 emulsions and therefore refer mainly to evaporation stars. In no case was an electron sensitive plate used.

§6. Determination of Attenuation Lengths in Lead

Recently an experiment has been carried out by Rosser & Swift (1951) to determine the attenuation lengths of the different sorts of nucleonic events which occur in electron sensitive emulsions. The purpose of the experiment was two-fold:-

(1) To compare the values of λ_A for different

which involve a <u>fast</u> charged primary particle with those formed by neutral primaries. The first group may be classified as $0p + 1p + 2p + \ldots$ and the second as $1n + 2n + 3n + \ldots$ Note that evaporation stars, i.e. 0n type stars, are excluded. For charged primaries under lead $\lambda_A = 405 \pm 31$ g.cm^{-2} whilst for neutral primaries the value falls to $\lambda_A = 260 \pm 34$ g.cm^{-2}. In air it is expected that the two values will be the same. The statistics clearly show that some stars below lead are produced by fast π-mesons. The numbers can be obtained from the data given in Tables 2 and 3. It is seen that the ratio of the events $0p + 1p + 2p + 3p + \ldots$ to all stars increases from $13.1 \pm 0.9\%$ for the air plates to $17.0 \pm 1.0\%$ for the lead plates, giving an increase of $3.9 \pm 1.3\%$. Rosser (1951) has made a rough calculation of the number of stars produced by fast π-mesons under lead, assuming that the primaries are absorbed as equation (3), that the rate of production of π-mesons is as given by the recent Bristol data, and that the value of λ_I for π-mesons is the length corresponding to the geometrical cross-section. With rough corrections for the loss of π-mesons by ionization and geometrical effects the result is about 5%. This compares favourably with the ob-

the lower atmosphere. A similar experiment carried out at Bristol seems to lead to the same result. Combining the two sets of results the ratio of charged to uncharged penetrating-shower primaries under 30 cm. of lead is (1.94 ± 0.32). This figure is greater than one with a probability of more than 99 per cent.

It is possible that part of the increase may be due to the production of penetrating showers by μ-mesons (George & Evans, 1950); it is, however, unlikely that more than 15 % can be due to this cause. Rather, it seems probable that events of the type 2p + 3p+... are penetrating showers produced by π-mesons. Several interesting mechanisms may be suggested.

(1) A π-meson may be elastically scattered inside a nucleus by a nucleon, both particles emerging with relativistic velocities. Such a process is energetically possible for a π-meson of energy greater than 2×10^{9} eV. The probability of the process occurring cannot be assessed since it involves the cross-section for scattering of π-mesons by protons, a quantity which is not known.

(2) A π-meson may give a large fraction of its energy to a nucleon in a nucleus and the nucleon then give rise to a penetrating shower (Heitler & Jánossy, 1950). According to Camerini et

in which an incident π-meson of kinetic energy 1085 MeV entered a nucleus and two π-mesons of energies 365 and 375 MeV emerged. Another case of a similar kind has been reported by Goldsack (1951) in which a negative particle (presumably a π-meson) of momentum (1,300 $\pm$ 130) MeV/c forms a star from which two particles at minimum ionization emerge. One of the emergent particles is negative and of momentum (1,000 $\pm$ 100) MeV/c; the other is at minimum ionization, but the momentum cannot be determined because of the shortness of the track. Application of the conservation laws to the event shows that it is unlikely the short track could be a proton.

It is important to find out if fast π-mesons can produce further π-mesons, for if the process is a probable one the secondary production of mesons within a nucleus will occur.

The total rate of stars after correction for background rates in the air plates was 9.0 ± 0.3/c.c./day and in the lead plates 2.96 ± 0.09/c.c./day.

The attenuation lengths of the different types of events are given in Table 4.

Table 4

Attenuation Lengths in Lead

Type of event	Attenuation length gm./cm^2
All stars	305 ± 7
$0p + 1p + 2p + \ldots$	406 ± 31
$1n + 2n + 3n + \ldots$	280 ± 34
$2n + 2p + 3n + 3p - \ldots$ (Penetrating showers)	380 ± 65

§7. The Comparison of λ_A and λ_I in Lead.

The attenuation length for all stars in lead is 305 ± 7 g.cm^{-2}, a value which agrees closely with those of George & Jason and Bernardini et al. (See Table 1).

In order to examine the possibility of events due to π-mesons, a comparison has been made of events

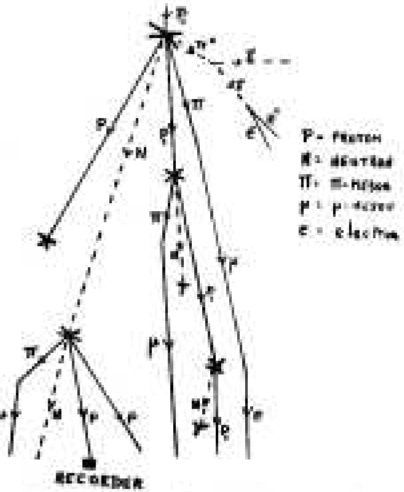

FIGURE 1—Nucleon Cascade in the atmosphere. (After Rossi 1951).

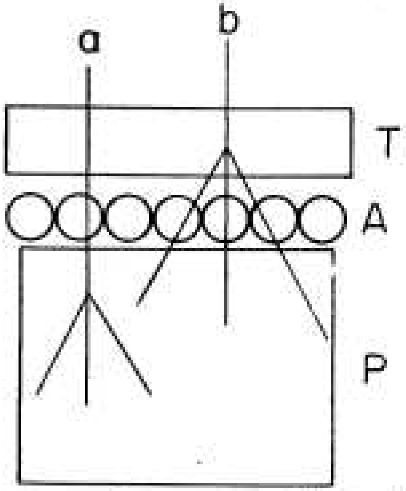

FIGURE 2—Counter arrangement for determination λ_L (After Rossi).

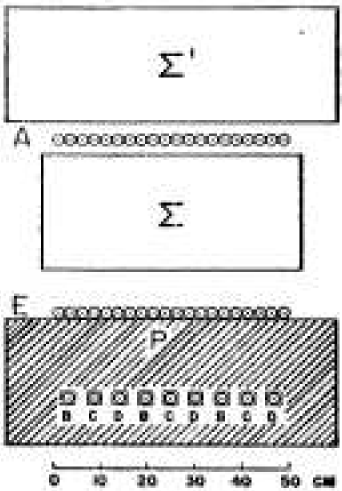

FIGURE 3—Cocconi's arrangement for determination of λ_L.

-served value of 3.9°/$_o$. Rosser and Swift have found eight cases of fast charged primaries which could be identified as π-mesons producing events of the types 0p or 1p or 2p. The number of stars due to fast π-mesons under 30 cm. lead is approximately equal to the number of σ-mesons produced by π-mesons at rest. The two are comparable since the probability of a π-meson stopping in the emulsion is of the same order as the probability of a fast π-meson giving rise to a nuclear interaction in the emulsion.

§6. Penetrating Showers under Lead.

Penetrating showers in the emulsion are defined as events for which $n_s \geqslant 2$, i.e. events of the types 2n + 2p + 3n + 3p + ... For all these events the value of λ_A is (380 ± 65) g.cm^{-2}. Taking for λ_I the value 200 g.cm^{-2} for corresponding events it is seen that the attenuation is 1.9 ± 0.3.

From Tables 2 and 3 the ratio of charged to uncharged primaries of penetrating showers (i.e. events of the types $n_s \geqslant 2$) is 0.86 ± 0.21 for the air plates and 1.67 ± 0.37 for the plates exposed under 30 cm. of lead. For nucleon cascades in air the ratio should remain constant and close to unity at all depths in al. (1950) a nucleon needs on the average an energy of 4×10^9 eV to give rise to a penetrating shower. Thus this process is only likely at rather high energies. An estimate of the contribution of this process can be made as follows. It has been shown that 4 % of all stars under 30 cm. lead are due to fast π-mesons. Less than 10 % of these mesons are likely to have energies greater than 4×10^9 eV. These π-mesons could lead to a small increase in ratio of charged to uncharged penetrating shower primaries. The maximum increase expected from this cause is estimated by Rosser & Swift at 20 %, which may be consistent with a ratio of $1.67 \pm .37$ but is hardly consistent with the value $1.94 \pm .32$.

(3) A π-meson may give rise to a shower of π-mesons by a multiple process in which the primary π-meson splits up into several mesons (Heitler & Janossy, 1950). This process might be expected to occur at energies of about 10^9 eV. The most likely event in the photographic emulsion would be a star of the type 2π. A possible case seems to have been found by the Bristol group. This is a $(3 + 2\pi)$ type star

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